Asset Prices When Investors Underestimate Discount Rate Dynamics

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Abstract

Underestimating discount rate volatility leads to asset pricing anomalies. Using analysts' return forecasts as proxies for subjective discount rates, I show that these forecasts exhibit systematically lower volatility than CAPM-based benchmarks, whose objective fluctuations negatively predict future returns, especially for high beta-volatility stocks. A misvaluation measure based on this underestimation significantly predicts cross-sectional CAPM alphas, while a tradable factor explains 12 prominent anomalies. These findings underscore discount rate underestimation as a unifying explanation for analysts' forecast errors and cross-sectional return predictability, linking recent evidence on aggregate subjective belief dynamics with firm-level mispricing.

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1 Introduction

A growing body of asset pricing research reveals a striking gap between investors' beliefs about asset prices and their observed behavior. One clear example is the markedly lower volatility in aggregate *subjective* discount rates relative to *objective* ones: while survey-based measures of expected returns or discount rates tend to be acyclical and show limited correlation with price ratios (De La O & Myers, 2021; Nagel & Xu, 2023), objective discount rates are strongly countercyclical and track aggregate price ratios closely (Cochrane, 2011; Koijen & Van Nieuwerburgh, 2011).

The broader asset pricing consequences of this "excess rigidity" remain underexplored, particularly at the individual stock level. If this rigidity also arises at the firm level, how does it affect cross-sectional pricing? These questions are further motivated by practical and pedagogical evidence suggesting investors often underestimate discount rate volatility.[1]

This paper proposes and tests the "Constant Discount Rate" (CDR) Hypothesis, asserting that some investors systematically underestimate discount rate fluctuations when valuing individual stocks. The findings indicate that such underestimation indeed manifests at the firm level and provides a unifying explanation for a broad set of well-known cross-sectional pricing anomalies.

I begin by further refining the CDR hypothesis, emphasizing its implications for both time-series and cross-sectional variations in stock valuations and returns. Specifically, the hypothesis encompasses two main dimensions: how CDR investors adjust discount rate estimates *in real time* and how they *perceive future* discount rate variability. While the former leads to direct, time-series predictions, the latter induces misvaluation differences across firms, generating cross-sectional predictions.

The time-series implications follow directly from the the hypothesis' core assump-

[1]Widely used valuation frameworks (e.g., (Damodaran, 2012; Koller, Goedhart, Wessels, et al., 2010)) typically assume constant discount rates, and surveys highlight limited practitioner focus on discount rate variability (Mukhlynina & Nyborg, 2016).

tion, leading to two key direct empirical predictions. First, because CDR investors underestimate discount rate variability, their subjective estimates should exhibit lower volatility over time compared to objective discount rates, especially among stocks with higher discount rate volatility.

I use sell-side analysts' return forecasts as my primary proxy for CDR investors' subjective discount rate estimates. These forecasts uniquely provide firm-level discount rate estimates across a broad cross-section of companies — an advantage rarely found elsewhere. To compare these estimates, I use conditional CAPM-based expected returns, estimated with a 90-day rolling beta, as my primary measure of objective discount rates.[2]

As a direct test, I compare the volatility of individual analysts' return forecasts with that of the CAPM-based estimates over the same calendar year. The results support the CDR hypothesis: objective discount rates are more volatile than subjective ones, indicating an underestimation of discount rate volatility, particularly among stocks with higher objective discount rate volatility. Quantitatively, among stocks in the highest volatility tercile, more than three-quarters of analyst-firm-year observations show lower subjective volatility. Even in the lowest volatility tercile, over 62% of the observations exhibit lower subjective volatility.

Analysts are relatively sophisticated professionals whose forecasts can influence market outcomes (Kothari, So, & Verdi, 2016). If enough investors exhibit similar underestimation as analysts, we should observe a negative relationship between time-series changes in objective discount rates and subsequent stock returns — a second time-series prediction of the hypothesis. Specifically, the CDR predicts that when objective discount rates rise (fall), the market under-adjusts, leading to temporary over-pricing (under-pricing) and thus lower (higher) future returns, especially for firms with higher discount rate volatility.

I test this prediction using firm-level panel regressions of next-month excess returns

[2] I consider alternative measures of objective expected returns in asset pricing tests in Appendix E.3.

on current conditional CAPM-based betas, controlling for firm and month fixed effects as well as various firm-level characteristics. The results again support the hypothesis: conditional beta strongly and negatively predicts next-month returns. Mirroring the results from the first prediction, this effect is most pronounced among firms in the highest (top-tercile) discount rate (CAPM-beta) volatility, where a 10% increase in conditional beta corresponds to a 3-basis-point drop in next-month excess returns.

Next, I examine the cross-sectional implications of the CDR hypothesis, which posits that firms with different fundamentals will be misvalued to varying degrees if investors underestimate discount rate volatility. Drawing on a valuation model that incorporates discount rate dynamics, I demonstrate that companies with both higher discount rate volatility and greater expected future cash flow growth (or longer cash flow duration) are especially susceptible to CDR-induced misvaluation. Intuitively, stable, well-established businesses (e.g., McDonald's) have relatively stable risk profiles, leaving less room for discount rate underestimation to matter, whereas cyclical or rapidly evolving firms (e.g., Tesla) do not. Furthermore, because underestimations compound over time, firms with extended payout horizons—those anticipating more distant cash flows—are particularly vulnerable to persistent misvaluation stemming from CDR.

As a first cross-sectional test, I examine whether a firm-level measure of CDR-induced misvaluation predicts stock returns. Specifically, I define a novel price-based measure as the difference between the implied cost of capital (ICC) and a conditional CAPM-based expected return.[3] The ICC — derived from a discounted cash flow formula — assumes a constant discount rate across all future periods, an approach previously cautioned in the literature (Hughes, Liu, & Liu, 2009) but directly aligned with the CDR hypothesis. Empirically, this measure increases with firms' discount rate volatility and cash flow duration, consistent with CDR.

[3] I do not use the difference between sell-side analysts' return forecasts and objective discount rates because analysts represent only a portion of market participants, making a price-based measure more appropriate.

The results strongly support the CDR hypothesis: this new misvaluation measure robustly predicts stock returns in the cross-section. Stocks in the highest (top-quintile) overvaluation group underperform those in the lowest (bottom-quintile) group by a Fama-French 5-factor alpha of 8% per year (t-stat = 5.30). This effect is economically large, persists for up to five years, and appears even within the S&P 500 universe. Moreover, the findings are robust to alternative specifications of discount rate models and are not driven by biases in analysts' cash flow forecasts.

Second, the CDR hypothesis posits that CAPM alphas stem entirely from investors' underestimation of discount rate volatility. In an ideal setting—where CDR is the only mispricing source and our misvaluation measure fully captures it — a factor-mimicking portfolio based on this measure would, in principle, explain *all* cross-sectional anomalies.

Empirically, I find that CDR indeed provides a powerful explanation for a large set of anomalies. Specifically, the misvaluation factor accounts for the CAPM alphas of 12 major anomalies, including 9 of the 11 studied by Stambaugh and Yuan (2017) (e.g., investment, profitability, beta, and idiosyncratic volatility). Crucially, it is derived primarily from analysts' forecasts and market prices, thereby avoiding any direct inclusion of anomaly-defining characteristics. Moreover, the characteristics used to define these anomalies predict future cash flow growth and/or discount rate volatility in ways consistent with CDR, consistent with the prescribed channel.

Finally, I examine whether the CDR hypothesis also applies to the cross-section of analysts' discount rate estimates. Indeed, analysts' consensus return forecast errors rise with both discount rate volatility and expected cash flow growth, reinforcing the connection between discount rate misperceptions and return anomalies.

Overall, the two time-series tests directly verify the CDR premise using both subjective discount rate estimates and individual stock prices. Meanwhile, the three cross-sectional tests indicate that underestimating discount rate volatility unifies various cross-sectional return anomalies and explains systematic variations in analysts' forecast errors.

This paper contributes to a growing literature that uses survey-based subjective expectations to understand asset prices (see, for example, Greenwood & Shleifer, 2014, as well as Adam & Nagel, 2023 for a survey). One strand of this literature focuses on short-term (Bouchaud, Krueger, Landier, & Thesmar, 2019) and long-term (Bordalo, Gennaioli, Porta, & Shleifer, 2024, 2019) cash flow expectations. These papers propose that investors' *cash flow* expectations are biased and show how such biases can help explain various time-series and cross-sectional pricing anomalies, holding subjective discount rates fixed.

Another strand of the literature investigates how *subjective discount rates* vary over time at the aggregate level (Dahlquist & Ibert, 2024; Jiang, Lustig, Van Nieuwerburgh, & Xiaolan, 2024). For instance, De La O and Myers (2021) and Renxuan (2020) find that subjective discount rates comove less with aggregate price–earnings ratios than do subjective cash flow expectations, while Nagel and Xu (2023) document that subjective discount rates exhibit insufficient cyclicality and volatility compared with their objective counterparts across a broad range of asset classes.

This paper bridges these two strands by investigating whether underestimation of *discount rate* dynamics also arises *at the firm level* and whether it can drive cross-sectional mispricing. In contrast to studies in the first strand, which focus on biased cash flow expectations, my approach starts with the assumption of *unbiased* firm-level cash flow expectations and explores whether investors' *discount rate* biases can generate cross-sectional return anomalies. Compared to the second strand, which centers on *aggregate* subjective discount rate time-series fluctuations, I examine whether similar underestimation manifests at individual stock level and its implications on cross-sectional pricing.

The paper also contributes to a large literature that seeks to explain cross-sectional asset pricing anomalies by relaxing the full-information rational expectation assumption (N. Barberis, Shleifer, & Vishny, 1998; Daniel, Hirshleifer, & Subrahmanyam, 1998; Hong

& Stein, 1999). Of particular relevance are recent works that jointly examine both asset prices and investors' subjective return expectations (Adam, Marcet, & Beutel, 2017; N. Barberis, Greenwood, Jin, & Shleifer, 2015; Collin-Dufresne, Johannes, & Lochstoer, 2017; Hirshleifer, Li, & Yu, 2015; Nagel & Xu, 2022), which primarily focus on aggregate time-series variations in asset prices and subjective expected returns. By contrast, this paper proposes a hypothesis about firm-level subjective discount rate dynamics and tests it using both time-series and cross-sectional data on individual stock pricing and subjective expectations.

Admittedly, this paper does not explore in detail why investors underestimate discount rate volatility at the individual stock level. Possible explanations include parameter learning (Li, Van Nieuwerburgh, & Renxuan, 2023; Nagel & Xu, 2022), bounded rationality stemming from the complexity of valuation (Simon, 1956), or behavioral biases (N. C. Barberis, Jin, & Wang, 2020). Understanding these mechanisms is an important avenue for future research.

The rest of the paper proceeds as follows. Section 2 develops the hypothesis and outlines the testable empirical predictions. Section 3 describes the data and empirical measures, followed by the results in Section 4. Finally, Section 5 concludes.

2 Hypothesis Development

The Constant Discount Rate (CDR) hypothesis posits that some investors ("CDR investors") underestimate discount rate variability when valuing individual stocks, leading to biased expectations and return predictability (i.e., "asset pricing anomalies").

The CDR hypothesis has two dimensions: it affects how CDR investors adjust discount rate estimates *in real time* and how they *perceive future* discount rate variability. The former produces direct, time-series predictions, while the latter drives misvaluation differences across firms, resulting in cross-sectional predictions. Below, we elaborate on these two aspects further and present their respective testable predictions.

2.1 Under-Adjusting Discount Rates in Real Time

First, because CDR investors believe discount rates vary less than they actually do, they under-adjust their discount rates in real time. This implies lower discount rate volatility for the subjective discount rate estimates of CDR investors, as stated in Prediction 1.

Prediction 1 (Volatility of Subjective vs. Objective Discount Rates)**.** *The subjective discount rate estimates by CDR investors for a firm will demonstrate lower volatility over time compared to the objective discount rates for the same firm within the same timeframe.*

The intuition behind the prediction is straightforward. Testing this prediction requires empirical measurements of the subjective discount rates of the CDR investors and the corresponding objective discount rates, as well as their volatility. I propose to use sell-side analysts' return expectations at the firm level as the main measure for subjective discount rate estimates of the CDR investors. The details of these measurements are outlined in Sections 3.2.1 and 3.2.2.

Beyond conducting direct tests on discount rate estimates, the hypothesis has direct predictions regarding the time-series relationship between objective discount rate variation and future stock returns, if there exists a substantial number of CDR investors in the market. Intuitively, the CDR investors' biased expectations will cause them to incorrectly value certain assets, leading to excessive buying or selling, and consequently, causing market mispricing. In Appendix A, I confirm this intuition through a model featuring a portion of investors with biases in their expected return estimates. The model shows these biases will be correlated with stocks' future CAPM alpha's.[4] Prediction 2 below presents the implications on individual stock returns when this bias is due to underestimation of discount rate volatility.

[4] The model features a multi-asset economy where a proportion of biased investors (CDR investors) interact with rational, risk-averse investors (arbitrageurs). This model is based on Kozak, Nagel, and Santosh (2018), and the expectation biases are in general form, applicable to any biased return expectations, not only the CDR-implied biases.

Prediction 2 (Discount Rate Variation and Future Returns). *A temporary increase (decrease) in a stock's objective discount rate should result in lower (higher) subsequent returns of the stock.*

The prediction is intuitive. Since CDR investors fail to update their discount rate estimates promptly, and they contribute to market pricing, stocks with a temporary increase in their objective discount rate are likely to experience temporary overpricing, resulting in lower subsequent returns.

The under-adjustment of estimates by CDR investors should have cross firm implications too. We should expect the effects of Prediction 1 and 2 to be stronger for stocks with relatively higher volatility in discount rates — when the actual discount rate variation for stocks is minimal, the potential for mispricing is also limited. I test these in the empirical section. Next, I discuss more cross-section predictions, but from the dimension of *under-estimating future* discount rate variation.

2.2 Under-estimating Future Discount Rate Variation

As the second aspect of the hypothesis, the CDR assumption posits that investors fail to fully account for the variability in discount rates over \textit{future horizons}. This underestimation of future discount rate fluctuations leads to biases in investors' current estimates of discount rates when valuing future cash flows of stocks. Since different stocks have different expected cash flows, the CDR hypothesis implies how these biases would vary with which firm-level characteristics.

To illustrate how the CDR leads to valuation biases that differ among firms, consider valuing an asset i that pays risky cash flows in the next two periods, $c_{i,t+1}$ and $c_{i,t+2}$. Suppose company i is expected to launch a new business line at the end of Period 1, significantly altering its risk profile in Period 2. This change necessitates different discount rates for each period, $\mu_{i,t}$ and $\mu_{i,t+1}$, respectively. The correct valuation should therefore be:

$$V_{i,t} = \frac{E_t(c_{i,t+1})}{\mu_{i,t}} + \frac{E_t(c_{i,t+2})}{\mu_{i,t+1}} \tag{1}$$

where $E_t(*)$ represents unbiased expectations.[5]

CDR investors, who do not account for the change in risk profile between periods, apply a constant discount rate $\widetilde{\mu}_{i,t} = \widetilde{\mu}_{i,t+1} \neq \mu_{i,t+1}$, resulting in the misvaluation:

$$\widetilde{V}_{i,t} = \frac{E_t(c_{i,t+1})}{\widetilde{\mu}_{i,t}} + \frac{E_t(c_{i,t+2})}{\widetilde{\mu}_{i,t}} \tag{2}$$

and a bias $b_{i,t} = \frac{\widetilde{V}_{i,t}}{V_{i,t}} \neq 1$.

For a different firm j, the magnitude of the bias may differ because its discount rate dynamics are different, i.e., $\widetilde{\mu}_{j,t} - \mu_{j,t+1} \neq \widetilde{\mu}_{i,t} - \mu_{i,t+1}$. For example, firm j may operate in a much more stable manner, with no plans to launch new business lines.

Despite its simplicity, this example mirrors real-life scenarios. Firm j, for instance, could represent a well-established consumer goods company with a stable risk profile and consistent demand. Consequently, applying relatively steady discount rates $\widetilde{\mu}_{j,t}$ for all future cash flow periods will closely align with its fair valuation. Conversely, Firm i might exemplify a company operating in a volatile industry with an untested business model, leading to larger valuation biases under the CDR assumption. Prediction 3 below formalizes this intuition on this cross-sectional prediction.

Prediction 3 (CDR-Induced Misvaluation and Cross-Sectional CAPM Alpha)**.** *An empirical measure of CDR-induced misvaluation, $\widehat{b}_{i,t}$, constructed based on the difference between the discount rates used by CDR investors and the objective discount rates, should negatively predict a stock's CAPM alpha.*

To test the prediction, I use the implied cost of capital (ICC) — a commonly used measure for estimating discount rates — as a proxy for CDR investors' discount rate

[5]Here I assume that expected cash flows and discount rates are uncorrelated for simplicity. Allowing for correlations would also introduce biases from the CDR assumption, as discussed in Hughes et al. (2009).

estimates. A crucial assumption in estimating ICC is that the discount rate remains constant across future cash flow horizons for all firms. This assumption mirrors the key mistake made by CDR investors, and when paired with a measure of objective discount rates, it captures CDR-implied misvaluation that varies across firms. I describe this measure in more detail in Section 3.2.4.

When assets distribute cash flows over more than two periods, the length of their payout horizons influences valuation biases. In Appendix B, I employ a valuation model for long-term assets with cash flows extending to infinite periods. The analysis reveals that, in addition to firms with higher discount rate fluctuations — as shown in the previous two-period example — companies with higher expected cash flow growth or longer payout horizons experience greater misvaluation due to CDR effects.

To understand the intuition behind this effect, it is important to recognize that CDR-induced biases influence discount rate estimates in every valuation period, accumulating over the entire payout horizon. Different stocks have varying effective payout horizons: firms with faster expected growth possess longer effective payout horizons (cash flow duration) than those with slower growth. Consequently, all else being equal, firms with higher expected growth accumulate these biases more significantly, resulting in more pronounced misvaluation.

Since many asset pricing anomalies are linked to these two fundamental characteristics, the CDR hypothesis can potentially explain a wide range of asset pricing anomalies.[6] I formalize this intuition in Prediction 4 below.

Prediction 4 (CDR-Induced Mispricing and Cross-Sectional Anomalies)**.** *A factor-mimicking portfolio constructed from CDR-induced misvaluation should explain a range of asset pricing anomalies. Furthermore, for these anomalies, firm characteristics that predict higher (lower) future abnormal returns should be negatively (positively) related to the firm's discount rate volatility and/or cash flow growth.*

[6] Gormsen and Lazarus (2023) show that firms with lower expected cash flow growth or shorter cash flow durations are common among various asset pricing anomalies, as in Prediction 4. A. Y. Chen and Zimmermann (2021) demonstrate that a large set of anomalies are related to firms' return volatility.

A factor-mimicking portfolio is a commonly used to tool in asset pricing. It is a long-short portfolio based on sorting stocks into CDR-induced misvaluation measure. This portfolio is expected to capture systematic mispricing in the cross-section of stocks.

Finally, beyond data on asset prices and returns, the CDR hypothesis should imply consistent patterns in data on subjective expectations. I test this in Prediction 5.

Prediction 5 (Expectation Errors and Firm Fundamentals)**.** *The errors in CDR investors' discount rate estimates increase with firms' expected cash flow growth and discount rate volatility.*

Together, these predictions provide a comprehensive framework for understanding how underestimating future discount rate variability can lead to both mispricing and cross-sectional asset pricing anomalies. The next section discusses the empirical methods used to test these hypotheses.

3 Data and Empirical Measurement

3.1 Data

I use the Institutional Broker's Estimate System (I/B/E/S) summary file for analyst earnings and price targets forecasts. I use COMPUSTAT annual data for balance sheet variables and CRSP for shares outstanding and share adjustment as well as price- and return-related variables. More detailed descriptions of the data sources are found in Appendix C.

The main dataset, merged between these two main sources of daata therefore differs from the more commonly employed CRSP-COMPUSTAT universe. It covers only about 40% of the number of firms and is restricted to larger firms. This reflects the fact that analysts typically focus on more established companies, with the results not including microcaps, which are less frequently covered by analysts.

3.2 Empirical Measures

3.2.1 Subjective Beliefs on Discount Rates

To test the CDR hypothesis regarding subjective expectations, I use sell-side analysts' forecasts as a proxy for investor beliefs about future discount rates. The key advantage of this dataset is that it provides estimates on return expectations at the *firm level*. Moreover, they seem to align with the behaviors expected from CDR investors, given evidence that they are systematically biased (Bradshaw, Brown, & Huang, 2013), correlate with cross-sectional anomalies (Engelberg, McLean, & Pontiff, 2019), and exhibit notable sensitivity to empirical risk proxies (Dechow & You, 2020).

I calculate subjective discount rates based on sell-side analysts' return expectations, using the formula: price targets divided by current prices, minus 1. The volatility of these return expectations serves as a proxy for the volatility of subjective discount rates. Appendix IA.C provides detailed information about the dataset and the construction of these measures.

I also define firm-level return expectation biases as the difference between the 12-month realized stock price returns and the sell-side analysts' consensus return expectations at the end of each quarter. The average bias for each firm is then calculated as the time-series average of these firm-level biases. These biases are primarily used in the tests for Prediction 5.

3.2.2 Objective Discount Rates

To measure objective discount rates, denoted by $\widehat{\mu}_{i,t}$, I use a conditional CAPM model. I calculate the expected return by multiplying each firm's beta by a constant market risk premium of 6.5%, which is the average realized excess return in our sample.[7]

$$\widehat{\mu}_{i,t} := \widehat{\beta}_{i,t} \times 0.065 \tag{3}$$

[7]The debate on the true expected return continues in the literature. I follow a similar approach to van Binsbergen and Opp (2019).

I estimate conditional beta, $\widehat{\beta}_{i,t}$, using rolling 90-day regressions based on daily stock and market excess returns. The rolling window method allows the beta to adapt to the most recent data, providing a dynamic measure of systematic risk. The choice of 90-day window is also slightly longer than the average frequency an analyst would update their forecasts, which is about two weeks. For each regression, at least 45 days of data are required; otherwise, the observation is excluded. To measure the volatility of objective discount rates, I calculate the rolling 250-day volatility based on the daily conditional beta estimates.

I use this measure of objective discount rates in testing both Prediction 1 and in constructing the firm-level misvaluation measure when testing Prediction 3. To verify results of the asset pricing tests are robust to different discount rate measure, Appendix E.3 also explores alternative ways to measure objective discount rates, such as multi-factor models.

3.2.3 Cash Flow Payout Horizon

For a firm's payout horizon, I use analysts' long-term growth expectations (LTG) as the primary proxy for equity duration. This follows from the approach of Gormsen and Lazarus (2023). The rationale is that higher long-term growth expectations imply a lower payout horizon, which corresponds to a longer cash flow duration. This reflects the idea that companies with higher growth potential may retain earnings longer, leading to extended future payouts. In addition to this, I consider anomaly returns based on the equity duration measures discussed by Dechow, Sloan, and Soliman (2004a), Weber (2018), and Gonçalves (2019) in the asset pricing tests.

3.2.4 Firm-level CDR-induced Misvaluation

To quantify the misvaluation caused by CDR investors, I construct a measure based on the CDR investment process discussed in Section 2. Intuitively, this measure should capture the difference between the time-varying objective discount rates and the discount rates assumed by CDR investors, who underestimate the its volatility.

To approximate CDR investors' discount rate estimates, $\Pi_{i,t}$, I use the firm-level implied cost of capital (ICC). The ICC is derived by projecting future cash flows and using historical prices under the assumption of a constant discount rate. To demonstrate how ICC inherently ignores discount rate variation, consider a simplified example, valuing a single cash flow $X_{i,t+k}$ received k periods from now. The true price of the asset at time t, given dynamic discount rates $\mu_{i,t+j}$, is:

$$P_{i,t}^{k} = \frac{E(X_{i,t+k})}{\prod_{j=0}^{k-1}(1+\mu_{i,t+j})}$$

In contrast, the ICC estimate $\Pi_{i,t}$ assumes a constant discount rate across all future periods and backs out the empirical estimate through the following pricing formula:

$$P_{i,t}^{k} = \frac{E(X_{i,t+k})}{(1+\Pi_{i,t})^{k-1}}$$

This implicit assumption means the ICC will differ from the true average discount rate whenever discount rates vary over time.

For empirical analysis, I use the ICC measure developed by Pástor, Sinha, and Swaminathan (2008), denoted as $\widehat{\Pi}_{i,t}$.[8] Diffrerent from the single cash flow example above, the ICC measure accounts for multiple future cash flow horizons but crucially assumes a constant discount rate for all horizons. This limitation has been noted in prior research (Hughes et al., 2009; Wang, 2015). Nevertheless, ICC remains widely used in practice and is featured in standard finance textbooks such as Damodaran (2012).

Estimating firm-level misvaluation requires six firm-level variables, one industry-level variable, and one aggregate variable: 1. Analysts' consensus forecasts for a firm's earnings for the current fiscal year (FY1), next fiscal year (FY2), and the fiscal year

[8]To ensure robustness, I also consider alternative models like Gebhardt, Lee, and Swaminathan (2001), which yield similar findings.

thereafter (FY3); 2. Analysts' consensus long-term growth forecast (LTG); 3. The payout ratio, calculated as the firm's total dividends from the previous year divided by its net income; 4. The firm's market beta; 5. The average LTG across 48 Fama-French industry classifications; 6. The long-term average GDP growth, ranging from 6% to 7% over the 35-year sample. Using these inputs, I compute the ICC, $\widehat{\Pi}_{i,t}$. More details on the estimation procedure are provided in Appendix D.

I define the CDR-induced misvaluation measure, $\widehat{\alpha}_{i,t}$, as the difference between the conditional CAPM-implied discount rate $\widehat{\mu}_{i,t}$ (as defined in Equation 3) and the ICC, $\widehat{\Pi}_{i,t}$:

$$\widehat{\alpha}_{i,t} = \widehat{\mu}_{i,t} - \widehat{\Pi}_{i,t} \tag{4}$$

This measure quantifies how much the discount rate assumed by CDR investors deviates from the time-varying discount rate, which reflects true market conditions. A positive $\widehat{\alpha}_{i,t}$ suggests that CDR investors are underestimating the firm's value, while a negative $\widehat{\alpha}_{i,t}$ indicates overestimation.

This is the primary variable used in my asset pricing tests for Prediction 3, which states this measure should be positively related to stocks' future CAPM alphas.[9] In Appendix E.3, I provide robustness checks using alternative measures of $\widehat{\mu}_{i,t}$.

An important point is that the estimation of firm-level misvaluation, $\widehat{\alpha}_{i,t}$, does not incorporate anomaly variables that we aim to explain in testing Prediction 4, other than $\widehat{\beta}_{i,t}$. Despite $\widehat{\beta}_{i,t}$ being mechanically *positively* correlated with the measure $\widehat{\alpha}_{i,t}$, the misvaluation measure still explains the 'low-beta' anomaly, where a firm's beta *negatively* predicts future returns. This indicates that the explanatory power of the misvaluation-based portfolio for a broad set of anomalies is not mechanically driven by including anomaly characteristics in its construction.

One potential concern is that analysts' cash flow forecasts, used to calculate the ICC, may be biased. In Appendix E.1 , I show that the results are robust when controlling for

[9] Equations (18) and (21) in the Appendix provide more clarification for this relationship.

ex-ante biases in these forecasts, using machine-learning methods to derive unbiased estimates, and even when including ex-post realized cash flow forecast errors. This finding is consistent with previous work by Hou, van Dijk, and Zhang (2012) and Wang (2015), which demonstrates that analysts' forecasts perform similarly to statistical models, particularly for large-cap firms.

One potential concern is that analysts' cash flow forecasts, used to calculate the ICC, may be biased. In Appendix E.1, I show that the results are robust when controlling for ex-ante biases in these forecasts, using machine-learning methods to derive unbiased estimates, and even when including ex-post realized cash flow forecast errors.[10] These results suggest that the cash flow forecast biases from analysts are not the primary driver for the asset pricing results I document.

4 Empirical Results

This section presents the empirical results for each of the testable predictions detailed in Section 2. Each subsection addresses the results for the five predictions, respectively.

4.1 Subjective vs. Objective Discount Rate Volatility

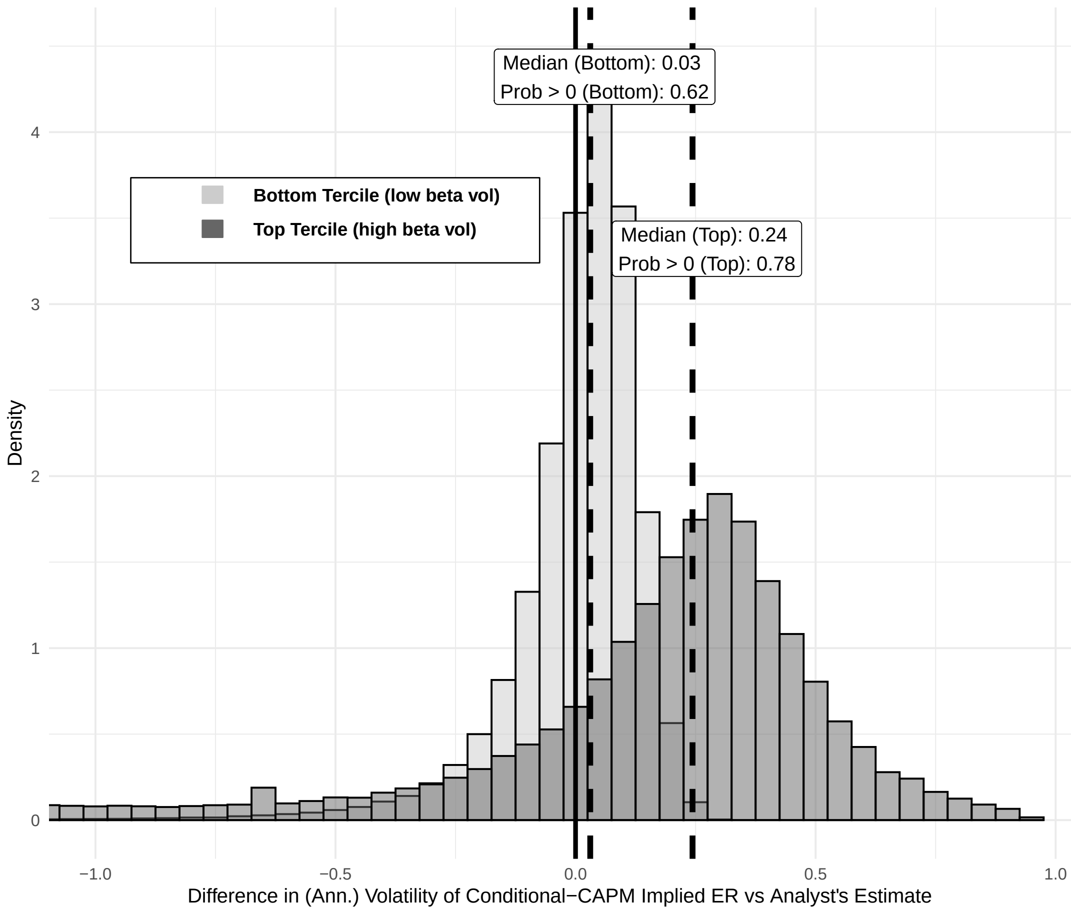
This subsection evaluates Prediction 1, which posits that analysts systematically underestimate the volatility of discount rates compared to the objective measures implied by the conditional CAPM. Subj. DR Vol. refers to the volatility of analysts' subjective discount rate estimates, while Obj. DR Vol. represents the volatility implied by the conditional CAPM. Comparing these measures allows us to assess whether analysts' subjective beliefs align with or deviate from objective risk-based expectations, providing evidence for the CDR hypothesis.

Specifically, at the end of each calendar year, I compute the difference between

[10] This finding is consistent with previous work by Hou et al. (2012) and Wang (2015), which demonstrates that analysts' forecasts perform similarly to statistical models, particularly for large-cap firms.

individual analysts' estimates of Subj. DR Vol. and the corresponding Obj. DR Vol over the same period. A higher difference indicates that analysts' own estimates of discount rates have lower volatility compared to the volatility implied by the conditional CAPM.

Figure 1: Comparison of Conditional-CAPM and Analyst Expected Return Volatility: High Beta Volatility vs. Low Beta Volatility Stocks



Notes: The figure shows histograms of the differences between the conditional-CAPM implied ER volatility and the analysts' ER volatility for stocks with low and high conditional-CAPM implied ER volatility. At each calendar year-end, stocks covered by analyst forecasts are ranked into terciles based on their conditional beta volatility, calculated using the past 252 days of a stock's conditional beta estimates, estimated using the rolling 90-day stock daily excess returns . The conditional-CAPM implied ER is computed as the conditional beta multiplied by 0.065, with ER volatility computed over 252 days and annualized. Analyst ER volatility is calculated by taking the time series of all return forecasts estimates issued by a given analyst within the year, computing the volatility of these estimates, and annualizing it. Both the analyst ER volatility and the conditional-CAPM implied ER volatility are winsorized at 1% and 99% before taking the difference.

To illustrate the results, Figure 1 presents histograms of these differences for stocks in the lowest and highest terciles of Obj. DR Vol., along with the median difference and the proportion of cases where Subj. DR Vol. exceeds Obj. DR Vol. Notably, the figure reveals that analysts systematically underestimate volatility, with this bias being more pronounced for stocks with higher Obj. DR Vol. For stocks in the lowest Obj. DR Vol. tercile, the median difference is 0.03, with Subj. DR Vol. exceeding Obj. DR Vol. in about 62% of the observations. For stocks in the highest Obj. DR Vol. tercile, the median difference is 0.24, with Subj. DR Vol. higher in 78% of the cases. These findings confirm that analysts systematically underestimate the volatility of discount rates, especially for stocks with higher objective volatility, as predicted by the CDR hypothesis.

Building on these results, we turn to Prediction 2, which examines how time-series variations in discount rates affect future stock returns, particularly for stocks with different levels of Obj. DR Vol.

4.2 Discount Rate Time-Series Variation and Future Returns

To test Prediction 2 , I run the panel regression:

$$r^{ex}_{i,t+1} = \alpha_i + \alpha_t + \lambda \widehat{\beta}_{i,t} + \delta' \text{Controls}_{i,t} + \epsilon_{i,t+1} \tag{5}$$

where $r^{ex}_{i,t+1}$ is stock i's next monthly excess return over the risk-free rate, and $\widehat{\beta}_{i,t}$ is the stock's "Cond(itional) Beta," estimated from the previous 90 trading days of excess returns. The regressions include firm (α_i), month (α_t) fixed effects and firm-level characteristics as controls. I run the regressions for the full sample, as well as for subsamples divided into high and low beta volatility terciles. According to the CDR hypothesis, we expect negative coefficient estimates for λ, particularly in the high beta volatility tercile, reflecting the mispricing caused by CDR investors' failure to adjust for discount rate dynamics.

Table 1 reports the results of the panel regression. In the full sample (Column 1), the coefficient on conditional beta ($\lambda = -0.668$) is significantly negative, indicating that a 10% increase in conditional beta predicts a 0.0668 percentage point drop in future monthly excess returns. This substantial negative relationship underscores the impact of discount rate variation on future returns, consistent with the CDR hypothesis that rising discount rates, unaccounted for by CDR investors, lead to lower future returns at individual stock level.

Table 1: Conditional Beta variation and future stock returns

	Fwd. 1-month Excess Return (pct)				
	Full Sample	High Beta Vol	Low Beta Vol	High Beta Vol	Low Beta Vol
	(1)	(2)	(3)	(4)	(5)
Cond. Beta	-0.668^{***}	-0.762^{***}	-0.273	-0.369^{***}	0.056
	(0.155)	(0.141)	(0.186)	(0.132)	(0.202)
Firm FE	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y
Controls	N	N	N	Y	Y
Observations	904,577	296,460	298,879	226,519	198,027
Adjusted R^2	0.118	0.101	0.219	0.114	0.234
Note:					*p<0.1; **p<0.05; ***p<0.01

The table reports the estimated coefficient λ from Regression 5 Model (1) estimates the regression for the full sample. Models (2) and (3) estimate the regression for firms whose conditional beta volatility (calculated based on the previous 252 daily betas) falls within the top and bottom terciles, respectively. Models (4) and (5) include firm-level characteristics as controls ($\text{Controls}_{i,t}$), which consist of the logarithm of market capitalization, book-to-market ratios (Fama & French, 1992), asset growth (Cooper, Gulen, & Schill, 2008), and profitability (Novy-Marx, 2013).

The effects are more pronounced among stocks with higher discount rate volatility. Specifically, in Column (2), for stocks in the top tercile of Obj. DR Vol., the coefficient on conditional beta becomes more negative ($\lambda = -0.762$) compared to the full-sample estimate. In contrast, the low beta volatility group (Column 4) exhibits a much weaker and statistically insignificant relationship ($\lambda = -0.273$). This pattern remains consistent even after including firm-level controls, although the coefficient for the high beta volatility group slightly decreases to ($\lambda = -0.369$). These results support the CDR hypothesis, suggesting that stocks with higher Obj. DR Vol. are more prone to mispricing due to

CDR investors underestimating discount rate volatility.

Overall, the empirical results in this subsection confirm that the temporary variation of Objective DR. leads to predictable future returns of individual stocks, consistent with Prediction 2. Additionally, stocks exhibiting higher Obj. DR Vol. demonstrate a more pronounced pattern of mispricing, which aligns with intuition of the CDR and the empirical evidence observed in the subjective expectation data shown in Figure 1.

4.3 CDR-Implied Misvaluation and the Cross-Section of CAPM Alphas

This subsection displays the results of Prediction 3. Prior to this, I will demonstrate the empirical characteristics of the CDR-implied isvaluation measure first before presenting the asset pricing results.

4.3.1 Empirical Properties of the Misvaluation Measure

I estimate the misvaluation measure following the steps in Section 3.2.4, which involves estimating ICC as detailed in Appendix D. Panel (a) in Table 2 presents the summary statistics for the resulting misvaluation measure and Table 10 in the Appendix provides detailed summary statistics for variables used to constructing the measure and the ICC.

As indicated Table 2a, most of the average firm-level misvaluations exhibit a negative sign. Although the absolute level of the measure is not essential for our cross-sectional results, this negative level provides insights about the shape of the objective term structure of discount rates. Specifically, it indicates the discount rates at longer horizons are greater than short-term horizon, and greater than the subjective discount rates assumed by CDR investors, who ignore the future dynamics. In fact, this upward-sloping term structure of objective discount rates are consistent with the alibration results from Ang and Liu (2004).

Table 2: Empirical Distributions of Key Variables for Misvaluation ($\widehat{\alpha}_{i,t}$)

The table presents the empirical distributions of the misvaluation measure defined in Equation (4). The data consists of firm-level data from 1986-01 to 2018-12. Empirical distributions in Panel (a) and the correlations in Panel (b) are based on average variable values over the entire time series for each firm. The term "ts.sd($\widehat{\alpha}_{i,t}$)" refers to the standard deviation of the quarterly misvaluation measure for each firm over its history. "N" represents the number of firms. In Panel (b), rank correlations are Spearman rank correlations. Panel (c) presents panel regression results based on firm-month data with month fixed effects.

(a) Empirical Distribution of $E(\widehat{\alpha}_{i,t})$

variable	mean	std	min	p25	median	p75	max	N
$E(\alpha_t^i)$	-0.080	0.072	-0.634	-0.098	-0.064	-0.040	0.059	7205.000
ts.sd(α_t^i)	0.026	0.018	0.000	0.015	0.023	0.033	0.200	7205.000

(b) Rank Correlation Between $E(\widehat{\alpha}_{i,t})$ and Firm Characteristics

$corr(\alpha_i, \sigma_{\beta_i})$	$corr(\alpha_i, g_i)$	$corr(\sigma_{\beta_i}, g_i)$
-0.357	-0.469	0.394

(c) Panel Regressions: Next Quarter Misvaluation ($\widehat{\alpha}_{i,t+3}$) and Firm Characteristics

	Dependent variable:		
		$\widehat{\alpha}_{i,t+3}$	
	(1)	(2)	(3)
$\sigma_{\beta_{i,t}}$	-0.096^{***}		-0.062^{***}
	(0.005)		(0.005)
$g_{i,t}$		-0.295^{***}	-0.253^{***}
		(0.014)	(0.014)
Observations	815,733	876,800	815,733
R^2	0.085	0.127	0.151
Adjusted R^2	0.085	0.126	0.151
Residual Std. Error	0.086 (df = 815390)	0.084 (df = 876363)	0.083 (df = 815389)
Note:			*p<0.1; **p<0.05; ***p<0.01

Furthermore, the measure shows significant cross-firm variation. Specifically, Table 2a indicates that the CDR-based misvaluation has a cross-sectional standard deviation exceeding 7% annually.[11] In comparison, there is considerably less variation over time,

[11]Note that the statistics in Table 2 may underestimate the extent of cross-sectional disparity in misvaluation of dynamically sorted portfolios. This primarily occurs because these portfolios rebalance by selecting stocks according to their misvaluation rankings each period, further increasing the dispersion

indicating the measure's high persistence. As depicted in the second row of Panel (a), the average quarterly volatility of firm-level misvaluation is 2.6%, which is about 4.6% annually.

This persistence in misvaluation is further supported by the high serial correlation coefficient. According to Table 10, an annual pooled panel regression reveals that $\widehat{\alpha}_{i,t}$ possesses an AR(1) coefficient of 0.948 (standard error 0.006, clustered by firm and year), indicating strong persistence.[12] A contributing factor to this persistence is the implied cost of capital, $\widehat{\Pi}_{i,t}$, which has an AR(1) coefficient of 0.92 based on quarterly data. These findings suggest that the CDR-induced market mispricing may also exhibit persistence, a topic explored further in Section 4.3.3.

Results in Panels (b) and (c) of Table 2 confirm the validity of the measure: the variations in the misvaluation measure are indeed influenced by the firms' discount rate volatility and expected cash flow growth, as predicted by the CDR hypothesis. Table 2b illustrates the rank correlation between the firm's average misvaluation and their average characteristics. Firms with higher discount rate volatility and higher expected cash flow growth tend to have lower average firm $\widehat{\alpha}_{i,t}$, aligning with the prediction that CDR investors more significantly misvalue these firms. Additionally, Table 2c presents results from panel regressions where the next quarter $\widehat{\alpha}_{i,t+3}$ is regressed on the firm-level characteristics. The R-squared and the estimated coefficients support the measure's validity and indicate that each of these two characteristics provides incremental explanatory power for the variation in $\widehat{\alpha}_{i,t}$.

4.3.2 Misvaluation Sorted Portfolios

This section evaluates Prediction 3 of the CDR hypothesis, which suggests that a stock's misvaluation measure, $\widehat{\alpha}_{i,t}$, should be a positive predictor of its CAPM alpha. To test this, I classify stocks by their misvaluation measure and analyze the average

after the rebalancing.

[12]This suggests a half-life exceeding 13 years for the misvaluation measure.

returns realized in various portfolios. Additionally, I compare the differences in ex-ante misvaluation estimates to the differences in realized CAPM alphas, and assess the economic significance of mispricing regarding its persistence and the market segments it influences.

Following the asset pricing literature (e.g., Fama and French (2015)), I sort stocks into quantile portfolios based on their misvaluation measure, $\widehat{\alpha}_{i,t}$. Portfolios are constructed at the end of June each year, using available data up to that point, and are rebalanced monthly based on value-weighting by market capitalization.[13] The holding period for each portfolio is 12 months. Table 3 summarizes the results. The ex-ante misvaluation measures on top of the table show the average of the the sorting variables (divided by 12) of each of the portfolios.

The results in Table 3 strongly support the hypothesis that misvaluation due to CDR biases is related to stock mispricing. First, Panel A shows that the most overvalued stocks ("High" misvaluation) experience significantly lower realized CAPM alphas. The difference in realized CAPM alpha between the most and least overvalued portfolios is 0.8% per month (9.6% annually), with a t-statistic of 5. Panel B confirms that this spread in alpha corresponds to a similar spread in realized returns, where the most overvalued stocks underperform by 0.7% per month (8.4% annually). The Fama-French 5-factor alpha spread is 0.69% per month (8.28% annually), also statistically significant.[14]

Moreover, as Panel C reveals, the ex-ante spreads in $\widehat{\alpha}_{i,t}$, which amounts to 12.91% (1.08% $\times$ 12), is driven primarily by the variations in the ICC estimates ($\Pi_{i,t}$), instead of the objective discount rates ($\mu_{i,t}$) in these portfolios. Indeed, the spread in the ICC estimates between the high and low portfolio amounts to -12.46% per ann, while the spreads in the objective discount rates is less than 1%. Additionally, the panel also

[13]Equal-weighted portfolio results, presented in E.4, show larger spreads in CAPM alphas.

[14]it is worth noting that all but the highest $\widehat{\alpha}_{i,t}$ portfolios exhibit negative realized CAPM alphas, despite value-weighted CAPM alphas theoretically summing to zero. This anomaly is driven by two factors: (1) firms with higher analyst coverage, which are more likely to have valid misvaluation measures, tend to exhibit lower returns on average (Diether, Malloy, & Scherbina, 2002; Hong, Lim, & Stein, 2000); and (2) stocks with higher misvaluation tend to be larger firms, which distorts the overall CAPM alpha distribution. Detailed data on firm characteristics are provided in Appendix C, Table 9.

shows that the most overvalued portfolios tend to consist of smaller firms. As pointed out in van Binsbergen and Opp (2019), anomalies appearing in only small firms and do not persistence over time have less real economic significance. Next, I examine next whether the effect is concentrated only among smaller stocks and how persistent the mispricing is.

Table 3: Pre-estimated Misvaluation ($\widehat{\alpha}_{i,t}$) Sorted Portfolios and Realized Average Stock Returns (1986-06 to 2019-12)

The table presents the statistics related to portfolios sorted based on the misvaluation measure created in Section 3.2.4. All numbers are expressed in percentages unless otherwise stated. Returns and alphas are based on monthly frequency.
Stocks are sorted into quantile portfolios based on the misvaluation measure $\widehat{\alpha}_i$ at the end of June each year, using the available information up to that point. Portfolios are rebalanced every month based on firms' market capitalization (value weighted). "Low" denotes the portfolio with lowest $\widehat{\alpha}_{i,t}$. "High-Low" denotes the excess returns of a portfolio that goes long on stocks with the highest $\widehat{\alpha}_{i,t}$ and short on those with the lowest $\widehat{\alpha}_{i,t}$.
Panel A presents the average monthly misvaluation, $\widehat{\alpha}_{i,t}/12$ associated with each of the portfolios
Panel B presents statistics related to portfolio returns. "mean ex.ret" are the monthly returns over three-month treasury rates; "SE" are standard errors which are shown in brackets. "SR" are monthly Sharpe Ratios. "FF-5 alpha" denotes Fama-French 5-factor alphas. "num_stocks" is the average number of stocks included in the portfolio over time.
Panel C presents characteristics (value weighted) associated with each of the portfolios. $\mu_{i,t}$ and $\Pi_{i,t}$ are the objective discount rates (beta times 6.4% and the ICC, respectively.

	Low	**2**	**3**	**4**	**High**	**High - Low**
Panel A: Ex ante Misvaluation vs. Realized Portfolio CAPM Alpha						
	Ex ante Misvaluation					
$\widehat{\alpha}_{i,t}/12$	-1.19	-0.61	-0.46	-0.32	-0.12	1.08
Realized Portfolio	Realized Portfolio CAPM Alpha					
CAPM alpha	-0.80	-0.39	-0.26	-0.09	0.01	0.80
SE CAPM alpha	(0.14)	(0.10)	(0.09)	(0.06)	(0.06)	(0.16)
Panel B: Realized Portfolio Return Statistics						
mean ex.ret	-0.03	0.27	0.33	0.48	0.66	0.70
SE ex.ret	(6.12)	(4.93)	(4.56)	(4.24)	(4.85)	(3.29)
SR	-0.01	0.05	0.07	0.11	0.14	0.21
FF-5 alpha	-0.63	-0.34	-0.41	-0.23	0.06	0.69
SE FF-5 alpha	(0.11)	(0.09)	(0.08)	(0.06)	(0.06)	(0.13)
Panel C: Portfolio Characteristics						
Mkt.Cap (Million)	15379.69	33550.85	38340.65	47129.95	88655.92	73276.23
$\mu_{i,t}$	7.14	6.22	6.26	6.29	7.59	0.45
$\Pi_{i,t}$	21.46	13.51	11.77	10.14	9.00	-12.46

4.3.3 Evaluating the Economic Significance of Mispricing

The Persistence of Misvaluation I find the CDR-induced misvalaution is persistent.

First, the persistence of the misvaluation measure, $\widehat{\alpha}_{i,t}$, is reflected in the low turnover of the misvaluation-based trading strategy. Table 4a shows that the average monthly turnover for both the long and short sides is around 2%, or less than 24% annually. Compared to strategies examined by Novy-Marx and Velikov (2015), this places the misvaluation strategy among the lowest turnover categories, similar to profitability-based portfolios and only slightly above size-based portfolios. This suggests that transaction costs are unlikely to erode the CAPM alpha generated by the strategy.

Moreover, misvaluation persists in stock prices well after portfolio formation. Stocks in the most overvalued portfolios continue to underperform, while those in the least overvalued portfolios outperform for extended periods. Table 4b shows that even after holding periods of over 60 months, the CAPM alpha of the High-Low portfolio remains statistically significant. The return spread decreases by 0.21% per month from the 12-month to 60-month holding period, which is consistent with the high persistence of the misvaluation measure as discussed in Section 4.3.1.

For value-weighted portfolios (Panel A), persistence primarily stems from the continued underperformance of stocks that are heavily overvalued due to CDR biases. In equally weighted portfolios (Panel B), both the long and short sides show sustained outperformance and underperformance, suggesting a potential interaction between firm size and the misvaluation measure. This interaction is explored further in the following subsection.

Table 4: The Persistence of Misvaluation

This table illustrates the persistence of misvaluation and its long-term effect on asset prices. A pooled panel regression based on annual data shows that $\widehat{\alpha}_{i,t}$ has an AR(1) coefficient of 0.948, with standard errors of 0.006 (clustered by firm and year), indicating a high level of persistence. Panel (a) reports the annualized turnover of portfolios sorted by misvaluation, where monthly turnover is multiplied by 12 to get annualized values. Panel (b) presents the CAPM alphas, both value- and equal-weighted, of portfolios sorted by $\widehat{\alpha}_{i,t}$, rebalanced at the end of June each year, from 1986-06 to 2018-12. The CAPM alphas are estimated by regressing the portfolio excess returns on market returns, using the universe of stocks for which $\widehat{\alpha}_{i,t}$ is available. This selection accounts for the tendency of stocks with higher analyst coverage to have negative CAPM alphas.

(a) Portfolio Turnover: Misvaluation-Sorted Portfolios

Portfolio	short-side	2	3	4	long-side	avg.long.short
ann.turnover	28.56%	36.44%	31.80%	27.43%	19.28%	23.92%

(b) Holding Period Returns of Misvaluation-Sorted Portfolios

	portfolio holding periods (in month)					
	12	24	36	48	60	72
Panel A: CAPM alphas of value-weighted portfolios						
low α^i	-0.612	-0.524	-0.524	-0.593	-0.461	-0.568
[t-stat]	[-4.212]	[-3.356]	[-3.323]	[-3.663]	[-3.399]	[-3.529]
high α^i	0.147	0.092	0.082	0.096	0.071	0.071
[t-stat]	[2.775]	[1.793]	[1.782]	[2.262]	[1.603]	[1.841]
High - Low	0.760	0.616	0.606	0.689	0.531	0.638
[t-stat]	[4.646]	[3.573]	[3.543]	[3.964]	[3.56]	[3.755]
Panel B: CAPM alphas of equal-weighted portfolios						
low α^i	-0.626	-0.588	-0.642	-0.639	-0.627	-0.632
[t-stat]	[-2.877]	[-2.719]	[-2.964]	[-2.943]	[-2.931]	[-2.91]
high α^i	0.384	0.368	0.352	0.355	0.343	0.351
[t-stat]	[3.376]	[3.215]	[3.263]	[3.293]	[3.278]	[3.395]
High - Low	0.984	0.929	0.954	0.969	0.930	0.943
[t-stat]	[5.827]	[5.47]	[5.569]	[5.638]	[5.773]	[5.524]

Misvaluation in Different Size Segments of the Market I find that CDR-induced mispricing is present even among the largest companies in the stock market, which is economically significant given that large firms dominate the market by capitalization. This implies that the mispricing channel suggested by the CDR hypothesis affects a substantial portion of the overall market, further demonstrating its economic significance.

Table 5 presents the results from a 3x3 double sort based on firm size and misvaluation. Among the smallest companies, the spread in CAPM alphas between the most and least overvalued portfolios is 1.08% per month (12.96% per year). Even in the largest segment of the market, where the average market capitalization exceeds $26 billion, the spread remains at 0.63% per month (7.56% per year), with a highly significant t-statistic close to 5.

In addition, I further assess the economic significance of the CDR-misvaluation in the S&P 500 universe, which consists of the largest U.S. companies, accounting for about 80% of U.S. market capitalization as of September 2020. The results in Table 15 of Appendix E.5 show that even within this universe, the spread in CAPM alphas between the most and least overvalued stocks is 0.39% per month (4.68% per year). Moreover, the Fama-French 5-factor alpha (FF-5) is higher, at 0.53% per month, due to the portfolio's strong negative loading on the small-minus-big (SMB) factor.

Table 5: Mean Excess Returns of Size and Misvaluation Sorted Portfolios (Value Weighted, 1986-06 to 2018-12)

This table presents the returns and characteristics for 3x3 portfolios independently sorted based on the misvaluation measure, $\widehat{\alpha}_{i,t}$, as defined in Equation (4), and market capitalization as of June of the previous year. All returns, alphas, and standard errors are reported on a monthly basis and expressed as percentages. "1_1" refers to the portfolio with the lowest market capitalization and the lowest misvaluation ($\widehat{\alpha}_{i,t}$), while "3_1" refers to portfolios with the highest market capitalization and the lowest $\widehat{\alpha}_{i,t}$. The portfolios are value-weighted monthly.
"SE" represents standard errors (in brackets). "mean ex.ret" denotes monthly returns in excess of three-month Treasury rates. "SR" refers to monthly Sharpe Ratios. "FF-5 alpha" denotes Fama-French 5-factor alphas. "num_stocks" represents the average number of stocks in the portfolio.
Post-formation portfolio characteristics include: "Π" (the implied cost of capital), "μ" (the average beta times 0.064).

stats	1_1	1_2	1_3	high-low.small	2_1	2_2	2_3	high-low.mid	3_1	3_2	3_3	high-low.large
mean ex.ret	0.33	0.92	1.41	1.08	0.08	0.58	1.07	0.99	0.05	0.36	0.6	0.56
SE ex.ret	(6.86)	(6.26)	(7)	(2.48)	(6.48)	(5.54)	(5.81)	(2.09)	(5.48)	(4.35)	(4.57)	(2.68)
SR	0.05	0.15	0.2	0.43	0.01	0.1	0.18	0.48	0.01	0.08	0.13	0.21
CAPM beta	1.26	1.14	1.25	-0.01	1.28	1.1	1.17	-0.12	1.14	0.94	1.02	-0.12
SE CAPM beta	(0.05)	(0.04)	(0.05)	(0.03)	(0.04)	(0.03)	(0.03)	(0.02)	(0.03)	(0.02)	(0.01)	(0.03)
CAPM alpha	-0.44	0.22	0.64	1.08	-0.73	-0.1	0.36	1.07	-0.65	-0.22	-0.02	0.63
SE CAPM alpha	(0.21)	(0.19)	(0.22)	(0.13)	(0.16)	(0.14)	(0.14)	(0.1)	(0.11)	(0.07)	(0.04)	(0.13)
FF-5 alpha	-0.39	0.15	0.56	0.95	-0.74	-0.23	0.25	0.97	-0.39	-0.39	0	0.39
SE FF-5 alpha	(0.1)	(0.09)	(0.13)	(0.13)	(0.08)	(0.06)	(0.07)	(0.1)	(0.1)	(0.07)	(0.04)	(0.12)
num_stocks	429.1	208.07	134.39		225.19	281.74	256.84		116.49	274.87	372.7	
ME (million)	219.77	245.87	264.56		912.54	972.94	1024.18		26798.68	46560.25	79038.81	
Π	0.17	0.12	0.1		0.18	0.12	0.1		0.17	0.12	0.09	
μ	0.06	0.06	0.07		0.07	0.06	0.07		0.07	0.06	0.07	

4.4 CDR Hypothesis and Cross-Sectional Anomalies

This section evaluates Prediction 4 of the CDR hypothesis. Specifically, I investigate the extent and the mechanism through which the CDR hypothesis can account for anomalies noted in the literature. To achieve this, I first construct a factor-mimicking portfolio based on the misvaluation implied by the CDR and subsequently select a set of anomalies to analyze.

Constructing the Misvaluation Factor To explain anomaly portfolio returns, I construct a factor-mimicking portfolio based on the misvaluation measure. I follow a procedure similar to Fama and French (2015). Specifically, I perform independent 3-by-3 sorts based on market capitalization and $\widehat{\alpha}_{i,t}$. Within each size tercile (small, mid, and large-cap stocks), I create a long-short portfolio by going long on stocks with the highest $\widehat{\alpha}_{i,t}$ and short on stocks with the lowest $\widehat{\alpha}_{i,t}$. The CDR factor is defined as:

$$\begin{aligned} CDR_t = & \frac{1}{3}(R_t^{high,small} + R_t^{high,mid} + R_t^{high,big}) \\ & - \frac{1}{3}(R_t^{low,small} + R_t^{low,mid} + R_t^{low,big}) \end{aligned} \tag{6}$$

Table 6 and Figure 2 provide the return statistics and cumulative returns of the CDR factor, respectively. The factor exhibits an annual volatility of 6.3% and a mean return of 10.8%. The majority of this return comes from the short leg, which holds the most overvalued stocks.

The cumulative return graph demonstrates that the strong performance of the CDR factor is not concentrated in any specific period over the past 33 years, further confirming the persistence of the misvaluation effect, as discussed in the previous section.

Choosing Anomalies I select portfolios sorted on profitability, asset growth, market beta, idiosyncratic volatility, and cash flow duration. I use these anomalies to demon-

Figure 2: Cumulative Returns of the CDR Factor (In Log Scale)

Notes: Sample period is 1986-07-01 to 2018-12-31. Stocks are sorted independently into 3 by 3 terciles based on market capitalization and $\widehat{\alpha}_{i,t}$ at the end of each June. The portfolios are rebalanced each month based on market capitalization. the CDR factor is the "high minus low" and constructed by

$$CDR_t = \frac{1}{3}R_t^{high} - \frac{1}{3}R_t^{low}$$

where $R_t^{high} = \frac{1}{3}(R_t^{high,small} + R_t^{high,mid} + R_t^{high,big} - 3R_t^f)$ and $R_t^{low} = \frac{1}{3}(R_t^{low,small} + R_t^{low,mid} + R_t^{low,big} - 3R_t^f)$.

Table 6: Return Statistics CDR Factor

Notes: Sample period is 1986-07-01 to 2018-12-31. Stocks are sorted independently into 3 by 3 terciles based on the market capitalization the previous June and the $\widehat{\alpha}_{i,t}$ at the end of each June. The portfolios are rebalanced each month based on market capitalization. The CDR factor is constructed by

$$CDR_t = \frac{1}{3}R_t^{high} - \frac{1}{3}R_t^{low}$$

where $R_t^{high} = \frac{1}{3}(R_t^{high,small} + R_t^{high,mid} + R_t^{high,big} - 3R_t^f)$ and $R_t^{low} = \frac{1}{3}(R_t^{low,small} + R_t^{low,mid} + R_t^{low,big} - 3R_t^f)$.

	CDR	low $\hat{\alpha}$	high $\hat{\alpha}$
Annualized Return	0.108	-0.004	0.110
Annualized Std. Dev.	0.063	0.208	0.190
Annualized Sharpe	1.704	-0.021	0.578

strate whether the channels suggested by the CDR are indeed at work, i.e. whether the characteristics associated with these anomalies are consistent with the two fundamental channels suggested by the CDR. I briefly discuss the motivation for choosing these anomalies here.

First, beta (see Fama & French, 1992) and volatility anomalies (Ang, Hodrick, Xing, & Zhang, 2006; Haugen & Heins, 1975) have garnered significant attention for challenging the traditional positive risk-return relationship and the CAPM framework. This has led to an extensive body of research attempting to explain these so-called "low-risk" anomalies (Black, 1992; Frazzini & Pedersen, 2014; Schneider, Wagner, & Zechner, 2020).

Second, I consider profitability (Fama & French, 2015; Hou, Xue, & Zhang, 2015; Novy-Marx, 2013) and asset growth anomalies (Cooper et al., 2008; Fama & French, 2015; Hou et al., 2015). Profitability positively predicts returns, while asset growth negatively predicts returns. Recent research shows these anomalies can explain much of the variation in cross-sectional returns (Fama & French, 2016; Hou et al., 2015). Theories explaining these anomalies are both behavioral (Bouchaud et al., 2019) and rational (Hou et al., 2015).

Finally, the cash flow duration factor (Dechow, Sloan, & Soliman, 2004b; Gonçalves, 2019; Weber, 2018) negatively predicts future stock returns. This factor is theoretically significant because it relates to the term structure of equity and links macro-finance theories to the time-series and cross-section of returns (Binsbergen & Koijen, 2015; Croce, Lettau, & Ludvigson, 2014; Lettau & Wachter, 2007; Santos & Veronesi, 2010).

Furthermore, to demonstrate the extent to which the CDR hypothesis can help us understand mispricing in the cross-section, I consider a broader set of anomalies as discussed in Stambaugh and Yuan (2017), along with the two composite mispricing factors. These anomalies are widely studied in both academic literature and practice due to their relevance to key financial theories and persistent empirical performance.[15] These factors, constructed from 11 anomalies, are shown to have strong explanatory power over numerous anomalies uncovered in the literature. I assess whether the CDR factor explains the returns of these composite mispricing factors as well as the individual anomalies they are based on.

4.4.1 Explaining Five Prominent Anomalies Through Two Fundamental Channels

The CDR hypothesis predicts that the CAPM alphas of individual assets should be fully explained by the CDR factor. To test this, I construct long-short anomaly portfolios based on five characteristics and regress their returns on the market excess return and the CDR factor, as defined in Equation (6):

$$R_t^i = \alpha^i + \beta_{CDR}^i CDR_t + \beta_m^i (R_t^m - R_f) + \epsilon_t^i \tag{7}$$

Under the CDR hypothesis, all alphas should be jointly zero:

$$H_0^{CDR}: \quad \alpha^i = 0 \quad \forall i = 1, \ldots, N$$

I test this hypothesis using the Gibbons-Ross-Shanken (GRS) test and also evaluate the

[15] The well-known "value" and "size" anomalies are not included because, in the post-1986-06 sample, they do not exhibit significant CAPM alphas.

Table 7: Anomalies Portfolio Alpha/Beta Before/After Controlling for CDR Factor

Sample period is 1986-07-01 to 2018-12-31. In Panel A, the GRS test statistics are presented, which test the null hypothesis that all α^is in Equation 7 are jointly zero under the CAPM or the model where market factor together with CDR factors are included. Panel B presents the tests for individual assets in Equation 7. Panel B1: the long-short anomaly portfolios are regressed on market excess returns over 3 month treasuries. Panel B2: long-short anomaly portfolios are regressed on (value-weighted) market excess returns and CDR factor defined in Equation (6). "beta" are measured using the last 5 years of monthly returns; "prof" are operating profitability defined in Fama and French (2015); "res.var" are measured using 60 days of daily returns and Fama-French 3-factor model; "asset.growth" are the change in total assets from the fiscal year ending in year t-2 to the fiscal year ending in t-1, divided by t-2 total assets at the end of each June using NYSE breakpoints; "cf.dur" are cash flow duration measure defined in Dechow et al. (2004a), a composite measure based on sales and book values. Except for the "cf.dur", all other portfolios are downloaded from Ken French's website and are long-short (value-weighted) portfolios constructed by subtracting the portfolio with the lowest decile of beta, var, res.var, asset growth by the highest decile and subtracting the highest profitability portfolio by the lowest profitability portfolio. Decile portfolios of "cf.dur" are downloaded from Michael Weber's website; the portfolios end on 2014-06-30 and are equally weighted.

Panel A: GRS. Test Results

Model	CAPM	Mkt + CDR
GRS-stat	5.422	1.003
P-value	0.000	0.416

Panel B: Tests on Single Anomaly Portfolios

	beta	res.var	prof	asset.growth	cf.dur
Panel B1: CAPM alpha of anomaly portfolios					
CAPM Alpha (%)	0.565	1.246	0.721	0.488	1.261
t-statistics	[2.288]	[3.777]	[3.593]	[2.909]	[4.124]
CAPM Beta	-1.046	-0.971	-0.456	-0.177	-0.432
t-statistics	[-18.8]	[-13.063]	[-10.077]	[-4.688]	[-6.471]
Panel B2: CAPM alpha of anomaly portfolios after controlling for CDR factor					
CAPM Alpha (%)	-0.129	-0.114	0.174	0.085	0.296
t-statistics	[-0.484]	[-0.337]	[0.799]	[0.466]	[0.926]
CAPM Beta	-0.983	-0.849	-0.406	-0.141	-0.345
t-statistics	[-18.013]	[-12.243]	[-9.137]	[-3.759]	[-5.393]
Loading on CER	0.748	1.467	0.590	0.434	1.036
t-statistics	[5.706]	[8.807]	[5.522]	[4.815]	[6.804]

individual alphas of the anomaly portfolios.

To ensure robustness and avoid replication errors, I use official sources for the anomaly portfolios. Specifically, I source beta, variance, and residual variance-sorted portfolios from Ken French's website and cash flow duration-sorted portfolios from Michael Weber's website.[16,17] Additionally, I validate my results using data and code provided by A. Y. Chen and Zimmermann (2022).[18]

The results, shown in Panel A of Table 7, strongly support the hypothesis. The GRS test statistic for the CDR factor is just over 1, with a p-value of 0.42, compared to 5.4 under the CAPM, indicating that the CDR factor fully explains the joint CAPM alphas of all five anomaly portfolios.

Examining individual anomalies in Table 7, the standalone portfolio alphas become statistically insignificant when the CDR factor is included. Notably, the loadings on the CDR factor vary across anomalies. Portfolios associated with idiosyncratic volatility ("res.var") and cash flow duration ("cf.dur") have the largest loadings, at 1.467 and 1.036, respectively, while asset growth and profitability ("prof.") exhibit considerably lower loadings.

The CDR hypothesis predicts these differences arise from variations in exposure to two fundamental channels: discount rate volatility and expected cash flows. To test this, I examine the pairwise correlations between the firm-level characteristics driving the anomalies and the two fundamental channels identified by the hypothesis.

Figure 3 reveals patterns consistent with the CDR hypothesis. Idiosyncratic volatility ("ivol") and equity duration ("equity.dur") are most correlated with beta volatility and long-term growth estimates, which proxy for the two fundamental channels. Moreover, "ivol" is more strongly correlated with beta volatility, while "equity.dur" is more corre-

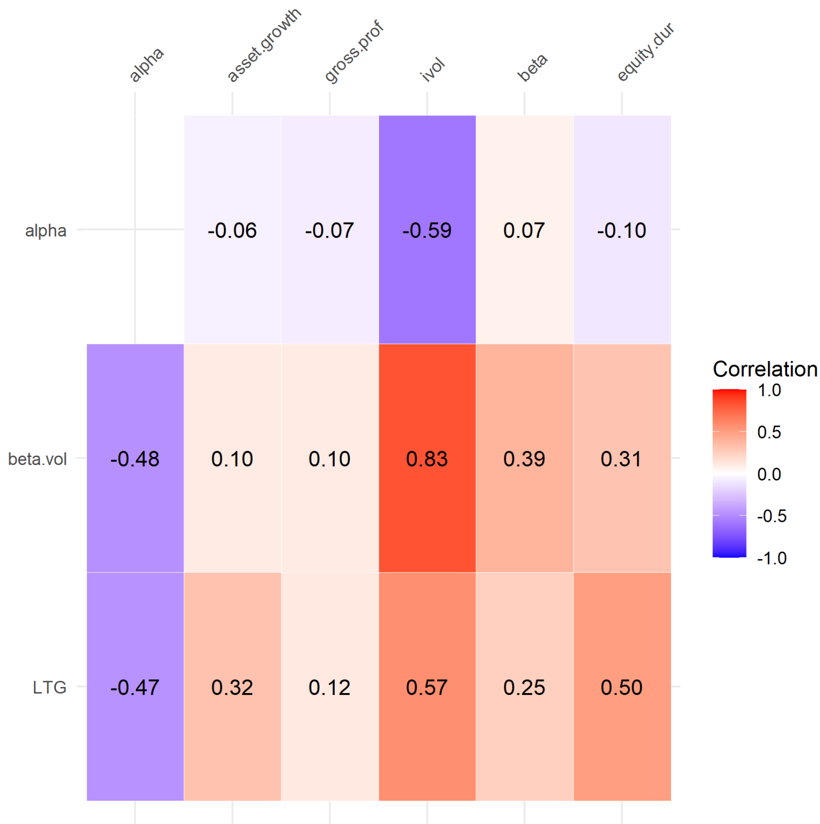
[16]Betas are estimated using five years of monthly returns; variances are calculated from the last 60 days of daily returns; residual variances are estimated using 60 daily returns and the Fama-French 3-factor model.

[17]Details are discussed in Dechow et al. (2004a) and Weber (2018).

[18]I re-run the related codes to replicate the anomalies from the website sources. I I use firm-level anomaly signals generated by the code of A. Y. Chen and Zimmermann (2022) for certain tests, including the correlation graph in Figure 3.

lated with long-term growth, aligning with intuition. These findings further validate the mechanism through which the CDR hypothesis explains anomaly returns.

Figure 3: Correlations Between CDR-implied Misvaluation (“alpha") and Firm-Characteristics

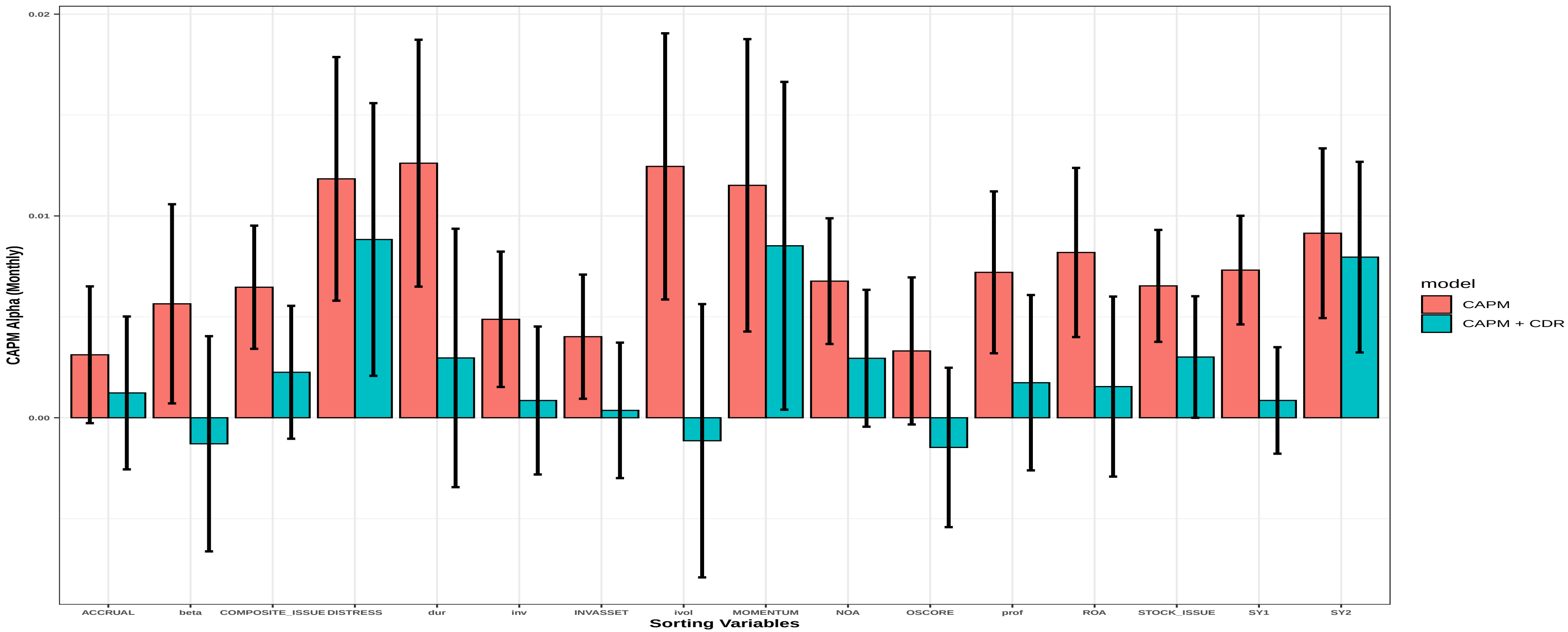


4.4.2 The Explaination Power of the CDR Factor: the Mispricing Factors of Stambaugh and Yuan (2017)

I further investigate the extent to which the CDR factor can explain return anomalies beyond the five examined earlier. Stambaugh and Yuan (2017) identify 11 anomalies in the literature and construct two mispricing factors (SY1 and SY2) that outperform the factor models of Fama and French (2015) and Hou et al. (2015) in explaining cross-sectional average returns. I test whether the single misvaluation factor derived from the CDR hypothesis can explain the CAPM alphas of these two mispricing factors and their underlying 11 anomalies, 9 of which were not covered in the earlier analysis.

Figure 4: CAPM Alphas of Long-Short Anomaly Portfolios Before and After Controlling for the CDR Factor

This figure plots the CAPM alphas for two mispricing factors from Stambaugh and Yuan (2017) alongside 11 anomalies used to construct these factors, as well as the duration, beta, and residual variance anomalies. Alphas are presented before and after controlling for the CDR factor.
The sample period spans July 1986 to December 2016. "CAPM" represents the intercept from regressions of long-short anomaly portfolio returns on market excess returns over the three-month Treasury rate. "CAPM + CDR" represents the intercept after including the CDR factor, as defined in Equation (6). Error bars indicate two standard deviations above and below the estimates. Long-short anomaly portfolios labeled in capital letters are sourced from Robert Stambaugh's website. "beta," "inv," "ivol," and "prof" are from Ken French's website, and "dur" is from Michael Weber's website. "ACCRUAL" refers to the accrual anomaly of Sloan (1996); "beta" is calculated using five years of monthly returns; "prof" denotes operating profitability as defined in Fama and French (2015); "ivol" is measured using 60 days of daily returns and the Fama-French 3-factor model; "inv" denotes asset growth as in Fama and French (2015) and Cooper et al. (2008); and "cf.dur" denotes cash flow duration as defined in Weber (2018). "COMPOSITE_ISSUE" refers to composite equity issuance from Daniel and Titman (2006), while "STOCK_ISSUE" refers to equity issuance as in Loughran and Ritter (1995). "DISTRESS" represents distress risk, as defined in Campbell, Hilscher, and Szilagyi (2008), and "OSCORE" is Ohlson's O-score from Ohlson (1980). "NOA" refers to net operating assets, as in Hirshleifer, Hou, Teoh, and Zhang (2004); "MOMENTUM" is defined in Jegadeesh and Titman (1993); "INVASSET" is the investment-to-assets ratio from Titman, Wei, and Xie (2013). "SY1" denotes the "MGMT" factor from Stambaugh and Yuan (2017), which includes net stock issues, composite equity issuance, accruals, net operating assets, asset growth, and investment-to-assets ratios. "SY2" denotes the "PERF" factor from Stambaugh and Yuan (2017), which includes distress, O-score, momentum, profitability, and return on assets.



The results show that the CDR factor provides substantial explanatory power for this broader set of anomalies, encompassing 16 in total. Figure 4 reports the CAPM alphas and standard errors for 14 anomalies (including the 11 from Stambaugh and Yuan (2017)) and the two mispricing factors, before and after including the CDR factor. In all cases, the CDR factor reduces the CAPM alphas. For 13 of the 16 anomalies, the CAPM alphas become statistically insignificant after accounting for the CDR factor. Notably, the CDR factor fully explains the first mispricing factor (SY1), which is primarily associated with longer-term anomalies, such as net equity issuance and accruals.

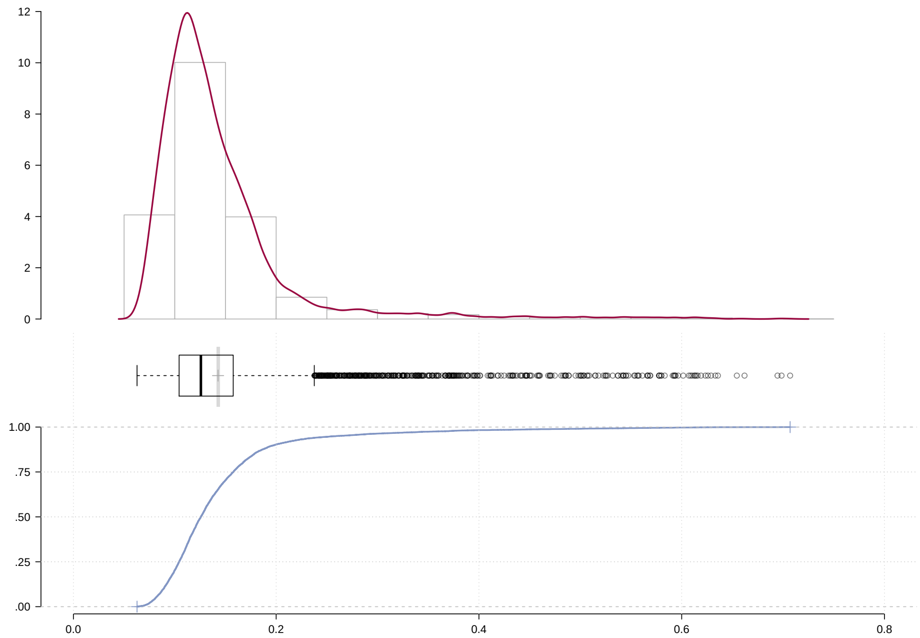
However, the CDR factor struggles to explain the second mispricing factor (SY2), which is linked to momentum and distress anomalies. These anomalies are characterized by short-term mispricing and high portfolio turnover, making them less compatible with the long-term nature of the CDR factor. This result is consistent with the hypothesis that the CDR factor primarily captures valuation errors stemming from underestimated long-term discount rate dynamics, while momentum and distress are driven by shorter-term market forces.

4.5 Expectation Errors and Firm Fundamentals

Finally, I test Prediction 5 of the hypothesis on cross-sectional expectation errors. The analysis begins with a discussion of the empirical properties of sell-side analysts' return forecast errors (described in Section 3.2.1), which serve as a proxy for CDR investors' biases.

Figure 5 illustrates the empirical distribution of firm-level return expectation biases from sell-side analysts, highlighting the mean and median (indicated by the bars). The distribution is right-skewed, with analysts' subjective return expectations being, on average, positive at the firm level. These patterns align with the properties of the misvaluation measure $\widehat{\alpha}_{i,t}$ discussed in Section 4.3.1.

Figure 5: Distribution of average firm-level analyst forecast errors of 12-month ahead returns



Notes: The top and bottom panel plot the empirical probability distribution function (PDF) and the cumulative distribution function (CDF) of average sell-side analysts' return forecast errors, respectively. The dark bar in the middle represents the median while the gray bar with a cross represents the mean. The x-axis is the value of the average biases while the y-axis denotes probability in percentage points. The forecast errors are constructed based on sell-side analysts' 12-month price targets subtracted by realized average returns. More details about how the return expectations are computed are documented in Appendix IA.C. Firm-level forecast errors are averaged over time to arrive at an average forecast error for each firm. The sample period is from 1999-Q2 to 2018-Q4.

This finding is consistent with prior research documenting the optimistic biases in analysts' price targets.[19] While much of the literature attributes this optimism to career incentives and conflicts of interest (Hong & Kubik, 2003), the CDR hypothesis provides an alternative explanation. According to CDR, analysts may systematically

[19]Studies documenting positive biases in analysts' forecasts include Brav and Lehavy (2003) and Engelberg et al. (2019).

overestimate returns because they fail to account for time-varying discount rates—an oversight stemming from cognitive biases rather than intentional motives.

Using these firm-level subjective expectation errors, I test the prediction that cross-sectional variations in analysts' return expectation biases are driven by discount rate volatility and expected cash flow growth, I estimate the following panel regression:

$$Bias_{i,t+1} = \alpha_t + \alpha_{ind,i} + b_1 \widehat{\sigma}_{\beta_i,t} + b_2 \widehat{k}_{i,t} + \delta' \text{Control}_{i,t} + \epsilon_{i,t} \tag{8}$$

where $Bias_{i,t+1}$ represents quarterly analysts' forecast biases; α_t and $\alpha_{ind,i}$ are time and industry fixed effects; $\widehat{\sigma}_{\beta_i,t}$ is the firm's 250-day beta volatility (used as a proxy for discount rate volatility); and $\widehat{k}_{i,t}$ represents analysts' long-term growth estimates, used as a proxy for payout horizon. Control variables include firm size (log market capitalization), book-to-market ratio, profitability, asset growth, and idiosyncratic volatility. Additionally, I run a cross-sectional regression using the time-series averages of the variables involved, without fixed effects, to provide a clearer interpretation of the R^2.

The CDR hypothesis predicts positive estimates for b_1 and b_2, indicating that firms with higher discount rate volatility and higher expected cash flow growth will exhibit greater expectation biases.

Table 8 presents the regression results. The first three columns display results for quarterly forecast biases ($fwd.1q.bias$), with columns (1) and (2) focusing on beta volatility and cash flow growth separately, while column (3) includes both variables along with controls. Columns (4) and (5) present results based on time-series averages, both with and without controls, to highlight the importance of beta volatility and cash flow growth in explaining the variation in biases.

The results strongly support the CDR hypothesis. The estimates for beta volatility are positive and highly significant across all specifications. In column (1), beta volatility has a coefficient of 0.284, and this effect remains robust even after controls are included in column (3), where the coefficient is 0.181. Similarly, cash flow growth is positive and significant, with a coefficient of 0.340 in column (2) and 0.230 in column (3). The time-series averages in columns (4) and (5) confirm these findings, showing that both beta volatility and cash flow growth contribute significantly to the variation in analysts' forecast biases. The coefficients of 0.812 and 0.760, respectively, in column (4) are associated with an adjusted R^2 of 16.5% for the model without controls. Even after including controls, the significance of both variables remains high, underscoring their importance.

Table 8: The Cross-Sectional Determinants of Average Firm-Level Forecast Errors of Sell-Side Analysts

The dependent variable is the next quarterly forecast bias ($Bias_{i,t+1}$) in columns (1) to (3), while columns (4) and (5) use time-series averages of forecast biases at the firm level. Beta volatility ($\widehat{\sigma}_{\beta_i,t}$) is the 250-day volatility of the firm's conditional beta, and cash flow growth ($\widehat{k}_{i,t}$) represents analysts' long-term growth estimates. Control variables include firm size (log market capitalization), book-to-market ratio, profitability, asset growth, and idiosyncratic volatility. Fixed effects include date and industry, and standard errors are clustered by firm and industry. The sample period is from 1999-Q2 to 2018-Q4.

	Dependent variable:				
	fwd.1q.bias			mean.bias	
	(1)	(2)	(3)	(4)	(5)
beta.vol	0.284***		0.181***	0.812***	0.379***
	(0.030)		(0.030)	(0.098)	(0.112)
cf.growth		0.340***	0.230***	0.760***	0.528***
		(0.048)	(0.041)	(0.173)	(0.132)
Control	No	No	Yes	No	Yes
Fixed Effects	Date+Ind.	Date+Ind.	Date+Ind.	No	No
Cluster S.E.	Date+Firm	Date+Firm	Date+Firm	Ind.	Ind.
Observations	69,455	60,746	60,746	2,223	2,223
Adjusted R^2	0.229	0.237	0.245	0.165	0.258

Note: *p<0.1; **p<0.05; ***p<0.01

These findings are consistent with the CDR hypothesis, which suggests that analysts' biases in return expectations are driven largely by firms' discount rate volatility and expected cash flow growth.

5 Conclusion

To bridge the disconnect between investors' subjective beliefs and objective asset pricing moments, this paper proposes and tests a novel hypothesis: some investors systematically underestimate the dynamics of discount rates when valuing individual stocks. This hypothesis is motivated by recent findings that investors' subjective return expectations are too rigid at the aggregate level and the evidence from textbooks and valuation practices that investors do not explicitly consider dynamics of discount rates when valuing individual stocks.

I find direct evidence supporting the hypothesis: analysts' subjective discount rate estimates are significantly less volatile than objective discount rate estimates, resulting the latter to negatively predict future individual stock returns. By formalizing this hypothesis, I also find this hypothesis can explain cross-sectional variation in stock returns and expectation errors. Constructing an intuitive empirical misvaluation measure, I show that the measure explains not only the CAPM alphas of individual stocks but also a wide range of well-known return anomalies. Overall, the paper shows evidence strongly supports the hypothesis, which has direct implications for cross-sectional stock pricing.

Future research could investigate potential mechanisms, such as investor learning processes or behavioral biases, to better understand the origins of this hypothesis.

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Appendix A Biased Return Expectations and Equilibrium Asset Prices: A More Formal Analysis

I study a multi-asset economy in which some investors with biased return expectations (CDR investors) trade with risk-averse rational investors (arbitrageurs). CDR investors take up $\theta \in (0, 1)$ share of the economy, so arbitrageurs are left with $1 - \theta$. Both of these investors live for two periods; in the first period they invest in the risky securities and have a risk-free rate r_f to maximize their terminal wealth. There are N risky assets, each of which pays a dividend of D_t^i for asset i in the next period. The number of shares outstanding of these risky assets is $x^* = (x^1, x^2, ..., x^N)'$, and risk-free assets are in unlimited supply.

Both CDR investors and arbitrageurs have the same utility function with the same risk-aversion coefficient, γ. The key difference is that the CDR investors have subjective return expectations, $\widetilde{E}(.)$, that are biased, or

$$\tilde{E}_t(R^i_{t+1}) = E_t(R^i_{t+1}) + b^i_t \tag{9}$$

In particular, the CDR investors solve the problem

$$\max_{\omega} \sum_{i=1}^{N} \omega^i P^i_t \left[\tilde{E}_t(R^i_{t+1}) - R_f\right] - \frac{\gamma}{2}\omega'\Sigma_t\omega \tag{10}$$

where

$$R^i_{t+1} = \frac{P^i_{t+1} + D^i_{t+1}}{P^i_t}$$

While the arbitrageurs solve the problem

$$\max_{y} \sum_{i=1}^{N} y^i P^i_t \left[E_t(R^i_{t+1}) - R_f\right] - \frac{\gamma}{2}y'\Sigma_t y \tag{11}$$

Denote $\omega^* = (\omega^1, \omega^2, ..., \omega^N)'$ and $y^* = (y^1, y^2, ..., y^N)'$ the optimal demand of the CDR investors and the arbitrageurs, respectively. The market clears, and we have

$$\theta\omega^* + (1-\theta)y^* = x^* \tag{12}$$

The equilibrium asset prices and expected returns are outlined in Proposition 1.

Proposition 1. *The multi-asset economy features biased investors and arbitrageurs whose return expectations are governed by Equation* (9) *and who solve optimization problems in* (10) *and* (11)*, respectively. With market clearing conditions* (12)*, the equilibrium asset price for asset i is*

$$P^i_t = \frac{1}{1 + R_f - \theta b^i_t}\left[E_t(P^i_{t+1} + D^i_{t+1}) - \gamma e^{i'}\Sigma_t x^*\right] \tag{13}$$

where e^i is a vector of zeros with 1 on the i^{th} entry. The expected return of asset i is

$$E_t(R^i_{t+1}) - R_f = \theta(-b^i_t + \beta^i_t b^M_t) + \beta^i_t\left[E_t(R^M_{t+1}) - R_f\right] \tag{14}$$

where $b_t^M = \sum_{i=1}^{N} \frac{x^i P^i}{\sum_j x^j P^j} b_t^i$ *is the market-level expectation bias of CDR investors,* $\beta_t^i = \frac{Cov_t(R_t^i, R_t^M)}{Var_t(R_t^M)}$ *(the CAPM beta in its usual definition), and* $R_t^M = \sum_{i=1}^{N} \frac{x^i P^i}{\sum_j x^j P^j} R_t^i$ *is the value-weighted market return.*

Proof. See Appendix IA.A. □

The results in Proposition 1 confirm the earlier intuition about how biases in the return expectation could cause mispricing in equilibrium. As shown in Equation (13), the more CDR investors in the economy, that is, the higher value of θ, the more serious the mispricing potentially becomes. Furthermore, when fixing the share of CDR investors, the higher the bias the CDR investors have for the return expectation of an asset, the higher its price and the lower its expected return, as shown in Equation (14). This is intuitive as the CDR investors will demand more of such an asset, leading to a lower expected returns.

Equation (14) reveals that the return expectation bias on the asset level as well as the market level together contribute to the non-zero CAPM alpha. This is intuitive as the CDR investors' irrational demand on the asset level would also lead to an equilibrium impact on the market level.

Appendix B Cross-sectional Variations in CDR-implied Biases and Stock Returns

I use a simple valuation model to demonstrate how CDR-implied biases would vary with firm-level fundamental characteristics. To focus the analysis on the consequence of under-estimating discount rate volatility, the model features a discount rate that is dynamic but uncorrelated with firms' expected cash flows. In Internet Appendix IA.B I show that this valuation model is consistent with an economy where the stochastic discount rate factor is driven by aggregate sentiment shocks that is uncorrelated with the aggregate consumption growth, similar in spirit to models proposed in De Long, Shleifer, Summers, and Waldmann (1990).

B.1 Average Valuation Bias and Firm Characteristics: An Example

Consider an asset i that provides a cash flow at a future time k, denoted $X_{i,t+k}$. To value this asset, it is necessary to account for the changing discount rates over future periods, $\{\mu_{i,t+j}\}_{j=0,\dots,k}$. Therefore, the asset should be valued as follows:[20]

$$V_{i,t} = e^{-\sum_{j=0}^{k-1} \mu_{i,t+j}} E_t(X_{i,t+k}). \tag{15}$$

Conversely, CDR investors erroneously employ a fixed discount rate $\widetilde{\mu}_i$,[21] leading to the valuation

$$\widetilde{V}_{i,t} = e^{-k\widetilde{\mu}_i} E_t(X_{i,t+k}). \tag{16}$$

The discrepancy between the two valuations is given by

$$b_{i,t} := \frac{\widetilde{V}_{i,t}}{V_{i,t}} = e^{-k(\widetilde{\mu}_i - \overline{\mu}_{i,t})}, \tag{17}$$

where $\overline{\mu}_{i,t} = \frac{\sum_{j=0}^{k-1} \mu_{i,t+j}}{k}$ represents the average of the future discount rates.

Equation (17) demonstrates that CDR investors experience valuation bias, represented as $b_{i,t} \neq 1$, when their own discount rates deviate from the average of future discount rates across various time horizons, i.e., $\Delta\mu_{i,t} := \widetilde{\mu}_i - \overline{\mu}_{i,t} \neq 0$. This discrepancy typically arises because CDR investors assume that future discount rates will remain constant.

For example, if the term structure of future discount rates is flat, meaning $\mu_{i,t+j} = \mu_{i,t}$

[20]To simplify the analysis, it is assumed that future cash flows are unaffected by fluctuations in the discount rate, allowing us to focus solely on the implications of the CDR assumption.

[21]The fixed discount rate assumption represents an extreme version of the hypothesis, suggesting underestimation.

for all j, then $b_{i,t} = e^{-k(\widetilde{\mu}_i - \mu_{i,t})}$. This implies that CDR investors will overvalue the asset when the actual discount rate temporarily spikes, consistent with Prediction 2 above.[22]

As I show in Appendix A,[23] when a fraction θ of investors have biased expectations the expected excess return for an asset i is given by:

$$E_t(R_{i,t+1}) - R_f = \theta(-b_{i,t}) + \beta_{i,t}\left[E_t(R^M_{t+1}) - R_f\right]. \tag{18}$$

Thus, the CAPM alpha of an asset i is inversely related to the expectation bias $b_{i,t}$ of the CDR investors:

$$\alpha_{i,t} = \theta(-b_{i,t}). \tag{19}$$

This leads to Prediction 3.

Besides, Equations (17) and (18) also suggest that the CDR-induced biases are related to firm-level characteristics, which gives rise to Prediction 4 and 5.

To see this, we consider the unconditional expectation of the biases based on Equation (17), assuming that future discount rates are i.i.d. and follow a normal distribution:

$$\mu_{i,t+j} \sim N(\overline{\mu}_{i,t}, \sigma^2_{\mu_i}), \quad \forall j = 0, \ldots, k \tag{20}$$

The average bias across different firms is given by:

$$E(b_{i,t}) = E\left[e^{-k(\widetilde{\mu}_i - \overline{\mu}_{i,t})}\right] = e^{kE(\Delta\mu_{i,t})}e^{\frac{k^2}{2}\sigma^2_{\mu_i}} \tag{21}$$

This equation shows the average bias on the firm level relates to the average difference between discount rates used by CDR investors and the objective rates, $E(\Delta\mu_{i,t})$. The sign and magnitude of this difference depend on the subjective discount rates used by CDR investors.

[22]In more general cases, such as an upward-sloping term structure of discount rates, $\overline{\mu}_{i,t}$ will be higher than $\mu_{i,t}$.

[23]In this appendix, we study a multi-asset economy where biased investors (CDR investors) interact with rational, risk-averse investors (arbitrageurs). This setup, based on Kozak et al. (2018), applies to any form of biased return expectations, not just CDR biases.

Furthermore, this equation confirms our intuition: on average, CDR-induced misvaluation varies across firms due to two main factors, discount rate volatility and payout horizon k. As the equation shows, even if CDR investors have no average bias in their rate estimation (i.e., $E(\Delta\mu_{i,t}) = 0$), underestimating volatility still causes cross-sectional differences in misvaluation and CAPM alphas due to k and σ_{μ_i}.

Specifically, higher discount rate volatility ($\sigma^2_{\mu_i}$) and longer payout horizons (k) both contribute to greater overvaluation by CDR investors. These results motivate the Prediction 4 and 5. Notice that although we consider a single cash flow occuring k periods into the future, similar logic applies to assets with multiple future cash flows, where the payout horizon k can be represented by the asset's cash flow duration. For such assets, the total value $V_{i,t}$ is the sum of the present values of all future cash flows k periods from now, $V^{(k)}_{i,t}$: $V_{i,t} := \sum_{k=1}^{\infty} V^{(k)}_{i,t}$. The timing of cash flows can be captured by a measure of cash flow duration (Dechow et al., 2004a; Weber, 2012), which is the weighted average time until the present value is recovered: $Dur_{i,t} := \sum_k^{\infty} \omega^{(k)}_{i,t} \times k = \sum_k^{\infty} \frac{V^{(k)}_{i,t}}{V_{i,t}} \times k$.

Appendix C Detailed Data Descriptions

In sum, the estimation of firm-level equity requires five firm-level variables, one industry-level variable, and one aggregate variable. The firm-level variables are: three analysts' consensus forecasts for a firm's earnings of the current fiscal year (FY1), the next fiscal year (FY2), and the fiscal year thereafter (FY3); one analysts' consensus long-term forecast (LTG); and one payout ratio, which is the ratio of the firm's previous year total dividend to the firm's net income. The industry-level variable is the average LTG based on 48 Fama-French industry classifications. The aggregate variable is the long-term average of GDP growth, which goes down from 7% to 6% over the 35 years in the sample. Based on these five inputs, I compute the implied cost of capital $q_{i,t}$ and the entire term structure of a firm's payout ratio $PB_{i,t+s}$ based on (22), which is a function of the last year's payout ratio and aggregate GDP growth rate and the $q_{i,t}$.

In the IBES monthly summary history file, I use analyst earnings per share (EPS) estimates for fiscal year 1, fiscal year 2, and fiscal year 3 (fpi = 1, 2, 3) and the long-term growth estimates to take full advantage of the term structure of analyst forecasts.[24] Furthermore, I require both fiscal year one and fiscal year two consensuses to be based on no less than three available analyst estimates and at least two estimates for FY3 consensus[25] in order to be included in the sample. I only use the latest monthly consensus estimates within each calendar quarter: March, June, September, and December to obtain firm-quarter consensus estimates. In addition, the firms included in the sample need to be U.S. firms whose reporting currency is in U.S. dollars. For the base case, I consider the median estimates as the consensus estimate, but my results do not change when using the mean estimates. To obtain estimates for total dollar earnings, the EPS estimates are multiplied by shares outstanding from daily CRSP data as of the date the EPS estimates were announced. In addition, I adjust for stock splits for the shares-outstanding data. To merge the IBES database with the CRSP database, I first match them using the 8-digit historical CUSIP. Additionally, I match firms whose ticker and/or company names are the same and those who have the same 6-digit historical CUSIP. In terms of timing, I match the quarterly IBES data with the monthly CRSP-COMPUSTAT merged by calendar quarter. In all asset pricing tests, I require the analyst estimates from the IBES summary file to be announced at least one quarter before the date that the returns are observed. Since the IBES summary file's statistical period is in the middle of each month, the analyst expectation information is lagged by about three months and two weeks.

To compute the payout ratio, I collect the common dividends (DVC) and net income (IBCOM) as well as the firm's historical industry SIC code from COMPUSTAT. If a

[24]Further horizons are available; however, the coverage is much poorer.

[25]The reason for using two FY3 estimates is that the coverage for FY3 is considerably poorer. My results are actually stronger when requiring three FY3 estimates; however, the average number of firms covered will be only 60% of the sample in the base case.

firm's net income is negative, I replace it with 6% of the asset value (AT). I winsorize the payout ratio so that it is also between zero and one. For other fundamental data and the price-related variables I use the CRSP-COMPUSTAT Merged (Annual) data. I include common shares (share codes 10 and 11) in the CRSP database traded on NYSE/AMEX and NASDAQ exchanges with the beginning-of-month prices above one dollar. When forming portfolios based on fundamental variables, I follow the convention in the literature (for example Fama and French (2015)), and lag the annual fundamental information of each firm for at least six months and assume that the information on all the firms' fundamental data is observed by end of June each year. Annual and monthly stock returns, as well as market prices and gross and net of dividends are obtained from CRSP and are adjusted for stock delistings. The market capitalization (ME) of a stock is its price times the number of shares outstanding, adjusted for stock splits, using the cumulative adjustment factor provided by CRSP, which is also used to compute a firm's total expected earnings and actual earnings.

Appendix D Estimating the Implied Cost of Capital

D.1 Methodology: The ICC Model of Pástor et al. (2008)

I follow the ICC model of Pástor et al. (2008) in estimating the implied cost of capital. L. Chen, Da, and Zhao (2013) details the way they calculate the ICC model in the cross section, and I therefore follow the procedure outlined in their appendix to estimate the ICC at the stock level.

1. Collect firm-level analyst earnings projections from the IBES monthly summary file. Include firm-level earnings projections at the end of March, June, September, and December for the current fiscal year (the next annual reporting date), the next fiscal year, and the long-term growth forecast (LTG);

2. Estimate the firm-level Implied Cost of Capital (ICC) model. This involves assum-

Table 9: Returns and Alphas of the Universe with Available Estimates of Misvaluation (Analyst's Forecasts)

Sample period 1985-07 to 2018-12. Monthly value-weighted excess returns of the universe with the available firm misvaluation measure $\hat{\alpha}_t^i$, or "vw.mkt.rf.analyst", are regressed on constant (Column 1), value-weighted excess returns of the market based on the CRSP universe (Column 2), and Fama-French five-factor returns downloaded from Ken French's website (Column 3).

	Dependent variable:		
		vw.mkt.rf.analyst	
	avg.ex.ret	CAPM.alpha	FF5.alpha
	(1)	(2)	(3)
mkt.rf		1.016^{***}	1.028^{***}
		(0.005)	(0.006)
smb			0.001
			(0.009)
hml			0.041^{***}
			(0.011)
cma			0.0003
			(0.016)
rmw			0.028^{**}
			(0.011)
Constant	0.005^{**}	-0.001^{***}	-0.002^{***}
	(0.002)	(0.0002)	(0.0002)
Observations	402	402	402
R^2	0.000	0.989	0.990
Adjusted R^2	0.000	0.989	0.990
Residual Std. Error	0.045 (df = 401)	0.005 (df = 400)	0.005 (df = 396)
F Statistic		$35{,}728.420^{***}$ (df = 1; 400)	$7{,}837.528^{***}$ (df = 5; 396)
Note:			$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

ing a firm-level long-term growth rate as well as a plowback rate (or $1-$ payout rate):

(a) Assuming

$$P_t = \sum_{k=1}^{15} \frac{FE_{t+k}(1-b_{t+k})}{(1+q_t)^k} + \frac{FE_{t+16}}{q_t(1+q_t)^{15}} = f(c^t, q_t) \tag{22}$$

where P_t is the stock price, FE_{t+k} is the earnings forecast k years ahead, b_{t+k} is the plowback rate ($1-$payout), and q_t is the ICC.

(b) Estimate FE_{t+k} :

i. FE_{t+1} and FE_{t+2} are proxied by the current fiscal year and the next fiscal year IBES analyst summary file data. $FE_{t+3} = FE_{t+2}(1 + LTG_t)$

A. Assuming the individual firm-level earnings growth rates to revert to industry growth forecast (LTG_t^{Ind}) by year $t+16$:

$$\begin{aligned} g_{t+k} &= g_{t+k-1} \times exp[log(LTG_{t+3}^{ind}/LTG_{t+3})/13] \\ &\quad \forall 4 \leq k \leq 15 \\ g_{16} &= g_t^{GDP}, \\ FE_{t+k} &= FE_{t+k-1}(1+g_{t+k}) \quad \forall 4 \leq k \leq 16 \end{aligned}$$

where g_t^{GDP} is the GDP growth rate using an expanding rolling window since 1947.

(c) Estimate b_{t+k} :

i. b_{t+1} and b_{t+2} are estimated from the most recent net payout ratio for each firm. The net payout ratio is common dividends (DVC in COMPUSTAT) to net income (item IBCOM). If net income is negative, replace it with 6% of assets.[26]

[26]Notice that about 50% of the firms do not pay dividends in the last year. As a result, during the first

ii. $b_{t+k}, 3 \leq k \leq 16$ is assumed to be

$$b_{t+k} = b_{t+k-1} - \frac{b_{t+2} - b_t^{ss}}{15} \tag{23}$$
$$\text{where } b_t^{ss} = g_t^{GDP} / q_t$$

(d) The q_t is then backed out by solving Eq. (22) and (23) together numerically. When there exist multiple roots, choose the root that is closest to the risk-free rate. Exclude any stock whose price is below one dollar. Winsorize the sample at 1% and 99%. Notice that by assuming the steady-state plowback ratio, we implicitly impose the constraint that

$$q_t \geq g_t^{GDP}$$

since in the steady-state, the plowback ratio can not exceed one.

two years, the plowback ratio is one. This does not mean that the projected earnings for the first two years have no impact on the estimation of the implied cost of capital q_t. Since FE_{t+k} are first calculated using the first two to three years of earnings projections together with the firm- and industry-level LTG, as long as any path during the first 15 years contains a non-zero payout ratio, the first three years of projections will have an impact on the estimation of the ICC.

D.2 Summary Statistics of the ICC and Input Variables

Table 10: Summary Statistics

(a) Empirical distributions of variables

	statistic	Pi	b_1_2	pb_7	EP_1	EP_2	EP_3	LTG	g_ind
1	mean	0.130	0.990	0.848	0.070	0.095	0.114	0.169	0.168
2	std	0.058	0.057	0.073	0.113	0.151	0.187	0.089	0.055
3	std cs	0.056	0.055	0.072	0.111	0.149	0.184	0.086	0.047
4	std ts	0.030	0.031	0.042	0.065	0.065	0.076	0.054	0.026
5	min	0.068	0.589	0.603	-0.153	0.004	0.014	0.040	0.048
6	p25	0.094	1	0.807	0.033	0.045	0.054	0.110	0.130
7	median	0.116	1	0.848	0.053	0.066	0.076	0.150	0.159
8	p75	0.144	1	0.896	0.076	0.090	0.104	0.200	0.195
9	max	0.428	1	0.993	0.876	1.216	1.512	0.500	0.351

(b) AR(1) coefficients

	variable	Pi	b_1_2	pb_7	EP_1	EP_2	EP_3	LTG	g_ind
1	AR(1)	0.920	0.943	0.882	0.897	0.950	0.956	0.893	0.946
2	std	0.005	0.009	0.004	0.008	0.005	0.005	0.005	0.010

(c) Correlations between variables

	Pi	pb_7	b_1_2	EP_1	EP_2	EP_3	LTG	g_ind
Pi	1	-0.746	0.023	0.611	0.747	0.784	0.409	0.336
pb_7		1	0.461	-0.352	-0.417	-0.434	-0.351	-0.284
b_1_2			1	-0.006	0.006	0.010	0.079	0.102
EP_1				1	0.873	0.821	-0.136	-0.107
EP_2					1	0.970	-0.070	-0.063
EP_3						1	-0.030	-0.042
LTG							1	0.511
g_ind								1

Note: Statistics are calculated over the whole sample. Firm-level variables are winsorized at 1% and 99%. “Pi” is the implied constant discount rate (ICC); “b_1_2” is the plowback ratio from the last year; “pb_7” is the implied plowback ratio in year seven. “Ek/P”, k = 1, 2, 3 are the fiscal year k earnings consensus estimates divided by the current market capitalization; “LTG” denotes long-term growth forecasts; “g_ind” denotes industry long-term growth estimates where industries are defined based on the 48 Fama-French classifications. In Panel 10a, “std” denotes the standard deviations for the variables over the entire sample, “std cs” and “std ts” are the average cross-sectional standard deviations over time and the time-series standard deviations over different firms, respectively. AR(1) coefficients are estimated by regressing the current value of the variable on its respective one-quarter lagged value based on the whole sample. Standard errors for the AR(1) coefficients are clustered by firm quarter.

Appendix E Robustness Checks

E.1 Return Predictability Controlling for Biases in Analysts' Earnings Forecasts

An alternative hypothesis to the CDR for the return predictability of the misvaluation measure is that biases in analysts' earnings forecasts—utilized in constructing the measure—may drive the observed results. Systematic biases in investors' cash flow expectations could inherently lead to return predictability. If analysts share these biases, as suggested in previous research ((Bordalo et al., 2024; La Porta, 1996; So, 2013; Van Binsbergen, Han, & Lopez-Lira, 2023)), the return predictability of the misvaluation measure might partly stem from predictable errors in analysts' cash flow (earnings) forecasts.

To test whether the predictability of the misvaluation measure is primarily driven by underestimating discount rate volatility, I conduct Fama-Macbeth regressions controlling for variables that capture cash flow forecast biases. Table 11 presents these results.

Column (1) shows that the misvaluation measure derived from the CDR hypothesis strongly predicts future returns, with a coefficient of 0.056, statistically significant at the 1% level. This suggests a 5.6% increase in future monthly returns for each unit increase in the misvaluation measure.

Column (2) introduces ex-ante analyst forecast biases and long-term growth (LTG) forecasts and their revisions. Ex-ante analyst biases, based on earnings forecasts for fiscal years 1 (FY1) and 2 (FY2) and machine-learning methods using large set of earnings predictors, negatively predict future returns (coefficient = -0.198), aligning with findings from Van Binsbergen et al. (2023). Consistent with the literature (Bordalo et al., 2024; La Porta, 1996), LTG and the last 12-month revisions in LTG also have a negative sign, although it is not statistically significant in our sample after controlling for the ex-ante analyst short-term cash flow bias measure.

Column (3) includes both the misvaluation measure and the ex-ante analyst earnings forecast biases for the short-term (FY1 and FY2) and the long-term (LTG and LTG revisions). The misvaluation measure's predictive power remains strong, with a coefficient of 0.056, suggesting that the CDR-based misvaluation is not primarily driven by predictable biases in analysts' earnings forecasts.

To alleviate the concern that ex-ante forecast biases are influenced by a particular econometric model, I also include as controls the future realized values of LTG revisions and short-term forecast errors in Column (4) and Column (5), respectively, besides additional controls—such as analysts' forecast revisions, forecast dispersion, gross profitability, and firm size. The misvaluation measure maintains a significant predictor for future returns, albeit reduced, predictive coefficient of 0.028 (significant at the 1% level) in Column (5).

Table 11: Return Predictability of Misvaluation Controlling for Earnings Forecast Biases

"fwd.1m.ex.ret" is the next month excess return over risk-free rate. "mis.val" is the misvaluation measure constructed in Section 3.2.4. "LTG" is the monthly analysts' long-term forecasts. "ex.ante.bias" is analysts' ex-ante predictable bias from their FY1 and FY2 consensus forecasts (Van Binsbergen et al., 2023). "FY1.fcst.error" and "FY2.fcst.error" are future realized forecast errors from analysts' FY1 and FY2 earnings consensus forecasts. Controls include the analysts' forecast revision (Haugen & Heins, 1975), the disparity in long and short term earnings forecasts (Da & Warachka, 2011) and firms' profitability (Novy-Marx, 2013), along with firms' market capitalization.

	Dependent variable:				
	fwd.1m.ex.ret				
	(1)	(2)	(3)	(4)	(5)
mis.val	0.056***		0.056***	0.054***	0.028***
	(0.006)		(0.007)	(0.009)	(0.009)
ex.ante.bias		−0.198**	−0.149*		
		(0.078)	(0.078)		
LTG		−0.001	0.007	0.032**	0.004
		(0.012)	(0.012)	(0.014)	(0.012)
delta.lag12m.ltg		−0.006	−0.005		
		(0.008)	(0.008)		
delta.fwd12m.ltg				0.138***	
				(0.014)	
FY1.fcst.error					−0.827***
					(0.036)
FY2.fcst.error					−0.199***
					(0.017)
Add.Controls	No	No	No	Yes	Yes
Observations	484,319	451,896	451,857	262,668	270,503
R^2	0.184	0.230	0.234	0.258	0.287
Note:					*p<0.1; **p<0.05; ***p<0.01

Note that part of the realized forecast errors may include information that was not observable when the misvaluation measure was constructed. Furthermore, future forecast revisions and forecast errors could also stem from shocks that affect both the realized discount rate forecasts and cash flow forecasts. Nevertheless, even though the coefficients on future revisions in LTG and future realized forecast errors for the FY1 and FY2 estimates are highly significant—and thus consistent with the notion that biases in cash flow expectations drive some portion of realized returns—the CDR-induced misvaluation measure still remains a strong predictor of future returns.

Overall, these findings suggest that the return predictability of the misvaluation measure is unlikely to arise from forecast biases in analysts' earnings projections, supporting the CDR hypothesis as the primary driver. These results are consistent with the conclusion from Wang (2015).

E.2 Predicting Analyst Return Forecast Biases Controlling for Earnings Forecast Biases

The CDR hypothesis predicts that both discount rate volatility and cash flow payout horizon should positively predict subjective return expectation errors in the cross-section. In Section 4.5, we tested this prediction using firms' conditional beta volatility and analysts' long-term growth forecasts (LTG) as proxies for discount rate volatility and cash flow payout horizon, respectively. The results, presented in Table 8, are consistent with the CDR hypothesis.

One concern with the positive relationship between analysts' LTG forecasts and future return expectation errors is the possibility of a common bias term in analysts' cash flow forecasts that could be driving this relationship, rather than the underestimation of discount rate dynamics.

To address this concern, I conducted additional robustness tests to show that biases in analysts' cash flow forecasts are not the primary drivers behind the results in Table 8.

Column (1) of Table 12 shows the results when replacing analysts' LTG with the equity duration measure introduced by Dechow et al. (2004a). This measure captures the cash flow duration of individual stocks, with higher equity duration indicating a longer cash flow payout horizon. Importantly, since equity duration is constructed using statistical models to forecast cash flow, it is free from the systematic biases that may be present in analysts' LTG forecasts. The results show a statistically significant positive relationship between equity duration and future analysts' return forecast errors, consistent with the CDR hypothesis. This suggests that biases in LTG are not the primary reason driving the results in Table 8.

In Columns (2), (3), and (4), I further include controls for analysts' ex-ante predictable biases, as well as analysts' future realized forecast errors for FY1 and FY2, respectively. The coefficients on beta volatility and equity duration remain positive and statistically significant, although the magnitude of the beta volatility coefficient decreases. These findings confirm that analysts' cash flow forecast biases are not driving the positive relationships between future return expectation errors and beta volatility or cash flow payout horizon. Thus, the prediction of the CDR hypothesis remains robust.

Table 12: Predicting Future Analysts' Return Forecast Biases Controlling for Earnings Forecast Biases

The dependent variable is the next quarterly forecast bias ($Bias_{i,t+1}$). "LTG" is the monthly analysts' long-term forecasts."Equity.Dur" is the equity duration measure from Dechow et al. (2004a). "ex.ante.bias" is analysts' ex-ante predictable bias from their FY1 and FY2 consensus forecasts (Van Binsbergen et al., 2023). "FY1.fcst.error" and "FY2.fcst.error" are future realized forecast errors from analysts' FY1 and FY2 earnings consensus forecasts. Controls include the analysts' forecast revision (Haugen & Heins, 1975), the disparity in long and short term earnings forecasts (Da & Warachka, 2011) and firms' profitability (Novy-Marx, 2013), along with firms' market capitalization.

	Dependent variable:			
	fwd.1q.bias			
	(1)	(2)	(3)	(4)
beta.vol	0.268***	0.206***	0.153***	0.153***
	(0.029)	(0.031)	(0.030)	(0.030)
Equity.Dur	0.006***	0.006***	0.007***	0.006***
	(0.001)	(0.002)	(0.001)	(0.001)
ex.ante.bias		1.819**		
		(0.836)		
FY1.fcst.error			−2.339***	−2.720***
			(0.406)	(0.257)
FY2.fcst.error			3.570***	4.044***
			(0.391)	(0.278)
Control	No	No	No	Yes
Qtr. & Ind. FE	Yes	Yes	Yes	Yes
Qtr & Firm Cluster S.E.	Yes	Yes	Yes	Yes
Observations	69,346	54,262	52,186	41,831
Adjusted R^2	0.231	0.262	0.343	0.373
Note:				*p<0.1; **p<0.05; ***p<0.01

E.3 Using Alternative Expected Return Models for Misvaluation Measure

To test the robustness of the return predictability of the CDR-induced misvaluation measure defined in Sec. 3.2.4, I examine alternative models for estimating dynamic expected returns when constructing the misvaluation measure. The primary measure used in the main analysis relies on the conditional-CAPM-beta times a constant risk premium as a proxy for the objective discount rate (expected return). Here, I replace this measure with four alternative expected return models commonly used in the literature and compute misvaluation as the difference between each alternative measure and the ICC.

The first alternative measure is the Fama-French six-factor model ("FF-6") from Fama and French (2018). I estimate rolling betas for each of the six factors using a 60-month window and multiply them by the factors' most recent monthly realized returns to proxy for expected returns. As a second measure, I use the Q-factor model proposed by Hou et al. (2015), which includes an expected growth factor in place of the momentum factor used in the FF-6 model. Rolling betas are computed similarly, and expected returns are derived by applying these betas to realized factor returns.

The third measure is the characteristics-based expected return model ("Char.Based.ER") from Lewellen (2014), which directly uses firm characteristics rather than time-varying betas when computing expected returns. Expected returns are estimated as the fitted values based on Fama-Macbeth regressions of firm-level future monthly realized returns on firm characteristics (size, book-to-market ratio, and momentum). Finally, I include the model developed by Lyle and Wang (2015), where expected returns are derived from regressions of future holding period (log) returns on firms' book-to-market ratio, return on equity, and prior-month mean squared returns.

Table 13 presents the predictive results for the misvaluation measures using each of these alternative expected return models. The findings indicate that the misvaluation measures strongly predict returns across all four models, with the two characteristics-based models yielding particularly strong results. This aligns with Lee, So, and Wang (2021), which concludes that characteristics-based measures are often more accurate proxies for cross-sectional firm-level expected returns.

Table 13: Return Predictability of Misvaluation Measure Based on Alternative Expected Return Models

"fwd.1m.ex.ret" is the next month excess return over risk-free rate. Independent variables are alterantive measures of the misvaluation, defined as the difference between expected return models (different each columns) and the ICC. The different expecgted return models are defined in the text.

	Dependent variable:			
	fwd.1m.ex.ret			
	(1)	(2)	(3)	(4)
FF-6	0.029***			
	(0.007)			
Q-Factor		0.027***		
		(0.007)		
Char.Based.ER			0.040***	
			(0.007)	
HPR				0.038***
				(0.007)
Observations	788,153	788,153	788,153	788,153
R^2	0.159	0.158	0.151	0.151
Note:				*p<0.1; **p<0.05; ***p<0.01

E.4 Equal-Weighted Portfolio Sorts

Table 14: Pre-estimated Misvaluation ($\widehat{\alpha}_t^i$) Sorted Portfolios and Realized Average Stock Returns (1986-06 to 2018-12, value weighted)

All returns, alphas, and their standard errors are expressed in percentages. Stocks are divided into quantile portfolios based on the misvaluation measure $\widehat{\alpha}^i$ at the end of June each year, using the available information up to that point. Portfolios are rebalanced with equal weights every month. "Low" denotes the portfolio with the lowest $\widehat{\alpha}_t^i$. "High-Low" denotes excess returns of a portfolio that goes long on stocks with the highest $\widehat{\alpha}_t^i$ and short on stocks with the lowest $\widehat{\alpha}_t^i$. "SE" are standard errors which are shown in brackets. "Mean ex.ret" are monthly returns over three-month treasury rates. "SR" denotes monthly Sharpe Ratios. "FF-5 alpha" denotes Fama-French 5-factor alphas. "num_stocks" us the average number of stocks included in the portfolio over time. "Ex Ante Misvaluation" denotes value-weighted portfolios $\widehat{\alpha}_t^i$ measured at each end of June. Their standard errors are measured using Newey-West methods based on four lags ("SE (NW-4)").

stats	Low	2	3	4	High	High - Low
Ex Ante Misvaluation	-1.8	-0.88	-0.67	-0.5	-0.3	1.5
SE (NW-4)	(0.19)	(0.32)	(0.21)	(0.18)	(0.15)	(0.12)
CAPM alpha	-0.63	-0.26	0.07	0.2	0.38	0.98
SE CAPM alpha	(0.22)	(0.19)	(0.13)	(0.11)	(0.11)	(0.17)
mean ex.ret	0.19	0.48	0.76	0.87	1.12	0.94
SE ex.ret	(7.22)	(6.28)	(5.56)	(5.23)	(5.91)	(3.31)
SR	0.03	0.08	0.14	0.17	0.19	0.28
CAPM beta	1.33	1.15	1.12	1.09	1.24	-0.08
SE CAPM beta	(0.05)	(0.04)	(0.03)	(0.02)	(0.03)	(0.04)
FF-5 alpha	-0.49	-0.29	-0.05	0.07	0.33	0.79
SE FF-5 alpha	(0.15)	(0.14)	(0.07)	(0.06)	(0.08)	(0.14)
num_stocks	456.68	453.78	456.09	456.09	453	

E.5 Misvaluation Sorted Portfolios Among S&P 500 Universe

Table 15: Misvaluation ($\widehat{\alpha}_{i,t}$) Sorted Portfolios and Realized Average Stock Returns for S&P 500 Firms (1986-06 to 2018-12)

This table presents statistics for portfolios sorted by the misvaluation measure $\widehat{\alpha}_{i,t}$ (as defined in Equation (4)) for firms in the S&P 500 universe. All values are expressed as percentages unless otherwise stated, with returns and alphas reported monthly.
Stocks are sorted into quantile portfolios based on the misvaluation measure, $\widehat{\alpha}_{i,t}$, as of the end of June each year, using available information up to that point. Portfolios are rebalanced monthly and value-weighted by market capitalization. "Low" refers to the portfolio with the lowest $\widehat{\alpha}_{i,t}$, while "High-Low" represents the excess return of a strategy that is long stocks with the highest $\widehat{\alpha}_{i,t}$ and short stocks with the lowest $\widehat{\alpha}_{i,t}$. "fwd_12m_alpha" refers to the average misvaluation measure 12 months after portfolio formation.
"CAPM alpha" is calculated by regressing portfolio excess returns on the universe of S&P 500 stocks with available estimates of $\widehat{\alpha}_{i,t}$.

stats	Low	2	3	4	High	High - Low
mean ex.ret	0.2	0.39	0.47	0.56	0.65	0.45
SE ex.ret	(4.78)	(4.53)	(4.26)	(4.34)	(5.13)	(2.97)
SR	0.04	0.09	0.11	0.13	0.13	0.15
CAPM beta	0.99	0.96	0.91	0.94	1.11	0.11
SE CAPM beta	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)
CAPM alpha	-0.29	-0.08	0.02	0.1	0.1	0.39
SE CAPM alpha	(0.1)	(0.08)	(0.07)	(0.07)	(0.08)	(0.15)
FF-5 alpha	-0.42	-0.4	-0.24	-0.2	0.11	0.53
SE FF-5 alpha	(0.1)	(0.09)	(0.08)	(0.07)	(0.09)	(0.15)
ME	43818.98	42726.38	54376.9	60586.93	110437.88	

Internet Appendix

IA.A A Proof of Proposition 1

Solving the first-order condition of (10) and (11), we have the optimal demands given by

$$\omega^* = \frac{1}{\gamma}\Sigma_t^{-1}\left[E(P_{t+1} + D_{t+1}) + B_t P_t - P_t(1 + R_f)\right] \tag{24}$$

where B_t is a diagonal matrix with biases b_t^i being on the i^{th} row and i^{th} column, and

$$y^* = \frac{1}{\gamma}\Sigma_t^{-1}\left[E(P_{t+1} + D_{t+1}) - P_t(1 + R_f)\right] \tag{25}$$

respectively.

Market clearing conditions imply that

$$\theta\omega^* + (1 - \theta)y^* = x^*$$

or

$$\theta\frac{1}{\gamma}\Sigma_t^{-1}\left[E(P_{t+1} + D_{t+1}) + B_t P_t - P_t(1 + R_f)\right] + (1 - \theta)\frac{1}{\gamma}\Sigma_t^{-1}\left[E_t(P_{t+1} + D_{t+1}) - P_t(1 + R_f)\right] = x^*$$

$$\theta B_t P_t + E_t(P_{t+1} + D_{t+1}) - P_t(1 + R_f) = \gamma\Sigma x$$

$$\left[(1 + R_f)I - \theta B_t\right] P_t = E_t(P_{t+1} + D_{t+1}) - \gamma\Sigma_t x^*$$

which leads to

$$P_t^i = \frac{1}{1 + R_f - \theta b^i}\left[E_t(P_{t+1}^i + D_{t+1}^i) - \gamma e^{i'}\Sigma_t x^*\right]$$

which is Equation (13) of Proposition (1).

The expected returns follow

$$\begin{aligned}
E_t(R^i_{t+1}) - R_f &= -\theta b^i_t + \gamma \frac{1}{P^i_t} e'_i \Sigma_t x^* \\
&= -\theta b^i_t + \gamma \frac{1}{P^i_t} e'_i Cov_t(P_{t+1} + D_{t+1}, P_{t+1} + D_{t+1}) x^* \\
&= -\theta b^i_t + \gamma Cov_t(R^i_{t+1}, P_{t+1} + D_{t+1}) x^* \\
&= -\theta b^i_t + \gamma Cov_t(R^i_{t+1}, (P_{t+1} + D_{t+1})' x^*) \\
&= -\theta b^i_t + \gamma Cov_t(R^i_{t+1}, R^M_{t+1}) P'_t x^* \qquad (26)
\end{aligned}$$

Now define the market-cap weight for asset i as

$$\omega^i_M = \frac{x^i P^i_t}{\sum_j x^j P^j_t}$$

and pre-multiply Equation (26) by the weights and sum over different assets to obtain

$$R^M_{t+1} - R_f = -\theta b^M_t + \gamma Var_t(R^M_{t+1}) P'_t x^*$$

which gives

$$\begin{aligned}
\gamma Var_t(R^M_{t+1}) P'_t x^* &= R^M_{t+1} - R_f + \theta b^M_t \\
P'_t x^* &= \frac{E_t(R^M_{t+1} - R_f)}{\gamma Var_t(R^M_{t+1})} \qquad (27)
\end{aligned}$$

Substituting Equation (27) into (26), we have

$$\begin{aligned}
E_t(R^i_{t+1}) - R_f &= -\theta b^i_t + \gamma Cov_t(R^i_{t+1}, R^M_{t+1}) \frac{E_t(R^M_{t+1} - R_f)}{\gamma Var_t(R^M_{t+1})} \\
&= \theta(-b^i_t + \beta^i_t b^M_t) + \beta^i_t \left[E_t(R^M_{t+1}) - R_f \right]
\end{aligned}$$

the last equation is Equation (14) in Proposition (1).

IA.B An Economy with a Persistent Sentiment-Driven SDF

I show the valuation example used in B.1 is consistent with an economy with sentiment investors, whose sentiment variation become a major source of priced risk that drives the time variation of the discount rate.

Consider a representative investor whose preferences depend on both consumption C_t and a sentiment state Z_t, with period utility:

$$u(C_t, Z_t) = \frac{C_t^{1-\gamma}}{1-\gamma} e^{-\lambda Z_t},$$

where $\gamma > 0$ is the coefficient of relative risk aversion, and $\lambda \neq 0$ governs how sentiment affects effective discounting or marginal utility.

The investor maximizes expected discounted utility and their first-order condition yields the stochastic discount factor from period t to $t+1$:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} e^{-\lambda(Z_{t+1} - Z_t)}.$$

where $\beta \in (0, 1)$ denotes the time-discounting parameter.

To simplify the analysis, assume aggregate consumption grows at a constant rate:

$$\frac{C_{t+1}}{C_t} = g$$

and the sentiment factor follows an AR(1) process:

$$Z_{t+1} = \phi Z_t + \varepsilon_{t+1}, \quad |\phi| < 1, \; E[\varepsilon_{t+1}] = 0,$$

with ε_{t+1} the shock to sentiment and is independent of consumption growth.

For any asset with payoff $X_{i,t+1}$ and price $P_{i,t}$, the pricing equation is:

$$P_{i,t} = E_t[M_{t+1} X_{i,t+1}] \implies 1 = E_t[M_{t+1} R_{i,t+1}],$$

where $R_{i,t+1} = X_{i,t+1}/P_{i,t}$ is the gross return. The risk-free rate $R_{f,tt}$ satisfies $R_{f,t}^{-1} = E_t[M_{t+1}]$.

To focus, we assume any asset's realized returns are only driven by shocks to a priced factor F_{t+1}, and this innovation only comes from the sentiment shock $\varepsilon t + 1$, i.e. $F_{t+1} := \varepsilon_{t+1}$, and

$$R_{i,t+1} - R_{f,t} = b_{i,t} F_{t+1} + e_{i,t+1},$$

where $e_{i,t+1}$ are i.i.d idiosyncratic shocks.

Given this setup, applying the pricing equation $1 = E_t[M_{t+1} R_{i,t+1}]$, we have the conditional expected return, or discount rate $\mu_{i,t}$ to be

$$\mu_{i,t} := E_t[R_{i,t+1}] = R_{f,t} + \beta_{i,t}(Z_t)\lambda_t(Z_t)Var(F_{t+1}),$$

where $\lambda_t(Z_t)$ reflects the time-varying price of sentiment risk. Both $\beta_{i,t}(Z_t)$ and $\lambda_t(Z_t)$ depend on the current sentiment state Z_t, yielding a time-varying, state-dependent expected return that is uncorrelated with aggregate consumption shocks or firms' cash flow shocks.

IA.C Measuring Analyst Return Expectations Using Analyst Price Targets

Firm-level analyst return expectations are constructed using a bottom-up approach based on analyst-level return expectations per analyst issuance.

I collect a single issuance of price targets from individual analysts' 12-month[27] price targets for individual firms from the IBES unadjusted database and then match it with the closing price from CRSP on the date the price target was issued[28] to compute return

[27]Other horizons are available, though the coverage is poor.

[28]In case the issuance date falls on a weekend, the last Friday prices are used. In case the issuance falls

expectations with price targets for individual firms. The expected returns are computed by dividing the analysts' price targets by the daily closing price on the day the estimate was issued and then subtracting one.[29] or

$$\mu_{i,f,d}^{A} = \frac{P_{i,f,d}^{A,12}}{P_{f,d}} - 1$$

where $P_{i,f,d}^{A,12}$ is the price target of analyst i for firm f, issued on day d. The superscript 12 denotes the 12-month ahead estimate. Notice this methodology ensures there is no mechanical relation between mean estimated expected returns and the level of prices. On each issuing date the analyst has the freedom to pick their own price target since they observes the prices.

Firm-level return expectations are constructed together with the stop file provided by IBES to ensure that individual estimates are not stale. IBES keeps track of the activeness of the individual estimates and provides a stop file for price targets.[30] I merge the point-in-time analyst-level expected return file with the stop file on price targets to exclude estimates that analysts and IBES have confirmed to be no longer valid. Furthermore, to avoid stale estimates, I further restrict the estimates to be no older than 90 days when entering mean consensus estimates.[31]

I construct weekly firm-level consensus expected returns by taking the mean of all active analyst-level forecasts, although using the median makes no discernible difference for the main results. I drop analyst-level estimates that are greater than

on a holiday, the previous business day closing prices are used.

[29]The same formula is used in Brav and Lehavy (2003) and Da and Schaumburg (2011).

[30]According to IBES, this stop file "includes stops applied to estimates that are no longer active. This can result from several events, e.g. an estimator places a stock on a restricted list due to an underwriting relationship or the estimator no longer covers the company. Prior to June 1993, actual stop dates did not exist in the archive files used to create the Detail History. An algorithm was developed to determine the date when an estimate became invalid if, for example, a merger between companies occurred or an analyst stopped working for a firm, etc. Estimate that are not updated or confirmed for a total of 210 days, the estimate is stopped."

[31]Engelberg et al. (2019) allows the estimates to be at most 12 months old, in case the estimates are not covered by the stop file, although the choice makes little difference for the main results, as verified in that paper's appendix.

five standard deviations away from the mean estimates, and I winsorize the entire analyst-level database by 1% and 99% before calculating the firm-level consensus. I take the mean of the available expected return estimates for each firm by the end of Saturday each week, or

$$\mu^A_{f,w} = \sum_i \mu^A_{i,f,w} / I_f$$

where I_f is the number of analysts for firm f at week w. For most of the application of the paper, I use firm-level return estimates based on monthly data, which is the consensus data on the last Saturday before each calendar month end.